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ABSCHÄTZUNG DER EIGENTÜMLICHKEITEN DER SPALTENNETZE NACH LANDOBERFLÄCHEN VER MESSUNGS ANGABEN

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ABSTRACT

Slope stability is a probabilistic phenomenon and as such should be analyzed with probabilistic methods. Fracture orientations alone, insufficiently model the rock mass. Other parameters describing the fracture set characteristics are required with distributions that can be sampled to fit mathematical stability models. Important considerations are length, spacing, dip, waviness angle, and overlap. These parameters are of major concern in design and are difficult to measure. Reasonable estimates of the characteristics can be used to provide distributions.

INTRODUCTION

Characterizing fracture properties for slope stability analysis or other surface works requires detailed geologic information on the major geologic structures with dimensions in the same order of magnitude as the slope and also on the less obvious rock fabric (the spatial orientation of all structural elements). Major geologic structures are considered as individuals in design as opposed to rock fabric which is treated statistically. Joint sets that individually are stable can combine with other sets to form a more complete failure path or a more complex failure mode.

Different failure modes can be postulated depending on the relationship between the geologic structure and the slope geometry. Three failure modes that have been observed in open pit mines serve to exemplify the modelling process. These modes are (1) plane shear on a single fracture, (2) step-path failure, and (3) three-dimensional wedge failure. In all of these modes, the characteristics of the fracturing play a most important role in the stability of the slope, because potential errors in the design are more likely to arise from the input parameters than from the method of stability analysis.

Slope failures are controlled by the joint set characteristics and their associated physical properties. Characteristics of particular importance are (1) length, (2) spacing, (3) waviness angle, (4) orientation, and (5) overlap (Figure 1). These variables are often distributed nonnormally.

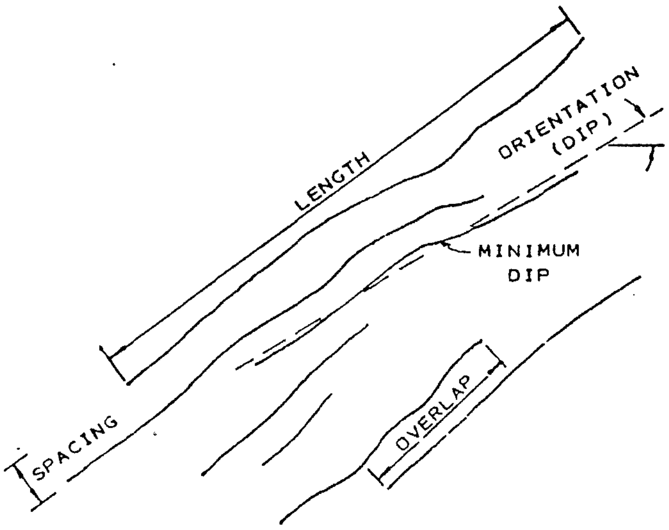


Figure 1. Joint Set Characteristics

Designing on average values for the variables accepts considerable risk because a mean value is a poor estimate of central tendency for nonnormal distributions. Höeg and Murarka (1974) present a design analysis in which the highest safety factor was calculated from mean values. In their design the highest probability of failure occurred when normally distributed input parameters were used. A probabilistic approach accounts for the variability in the input parameters if representative distributions can be obtained for the fracture properties.

Included in this discussion are methods of obtaining joint set characteristics by detailed mapping and some typical distributions that have been found.

DATA COLLECTION

Joint set characteristics must be measured to obtain the necessary distributions used in the design analysis. Statistical methods provide an estimate of joint set properties from limited observations.

Whitten (1966) discussed the difficulties of representative sampling in data collection and how these difficulties related to statistical analysis. He defines a target population as one that includes all joint sets of interest and their associated properties from which inferences or conclusions will be made during the design. In a proposed or relatively new open pit this target population is usually the fractures at depth that are unobservable except by boreholes. Commonly, the unobservable target population must be represented by surface mapping at limited exposures. The sampled population includes all of the joints potentially measurable. The sample is the particular joint that is actually measured and described.

Unless the target population and the sampled population are the same, the step to go from sampled population to target population is based on geology and engineering judgment and not statistical methods (Whitten, 1966). A bias may be introduced because of the limited size of the exposure where the joints are mapped. Actually, the measured joints may have a different structural relationship at depth than at the surface or have properties significantly different from the target population. Associated with this bias is a "window" problem where the size of the exposure limits the mappable size of observed joints and the maximum mappable spacing of joints within the set.

Another difficulty in sampling was considered by Terzaghi (1965). A "blind zone" occurs where fractures with strikes parallel to the mapping direction are overlooked or sampled to a lesser degree than fractures with strikes more normal to the mapping direction.

On a detail line survey it may be difficult to get a sufficient number of observations to describe the less obvious sets, although these sets may have significant implications in a design. Robertson (1970) has suggested that 100 observations per joint set should be obtained to define the joint set density with 95 percent confidence to within ± 20 percent of the true value. He further suggested that the same 100 observations per set be used as a guide to the reliability of other properties for the set. This large number of observations per set is an ideal that is rarely achieved in practice. Generally 150 observations are taken per detail line to define the rock fabric for porphyry copper desposits. The number of observations required depends on the fracture intensity and number of sets (Pincus, 1951; McMahon, 1968; Spencer, 1969; Call, 1972; Savely, 1972).

Because a detail line survey may give insufficient data on important joint set characteristics, special methods may be needed to supplement the detail line information.

Detail Line Survey

The detail line method is a systematic spot sampling technique (Halstead and others, 1968; Piteau, 1970a; Piteau, 1970b; Call, 1972; Savely, 1972). Ideally, the sample sites would be randomly selected and the lines would have random orientations to assure that the data would be representative of the target population. In practice, the sites are determined by availability and accessibility of the exposure.

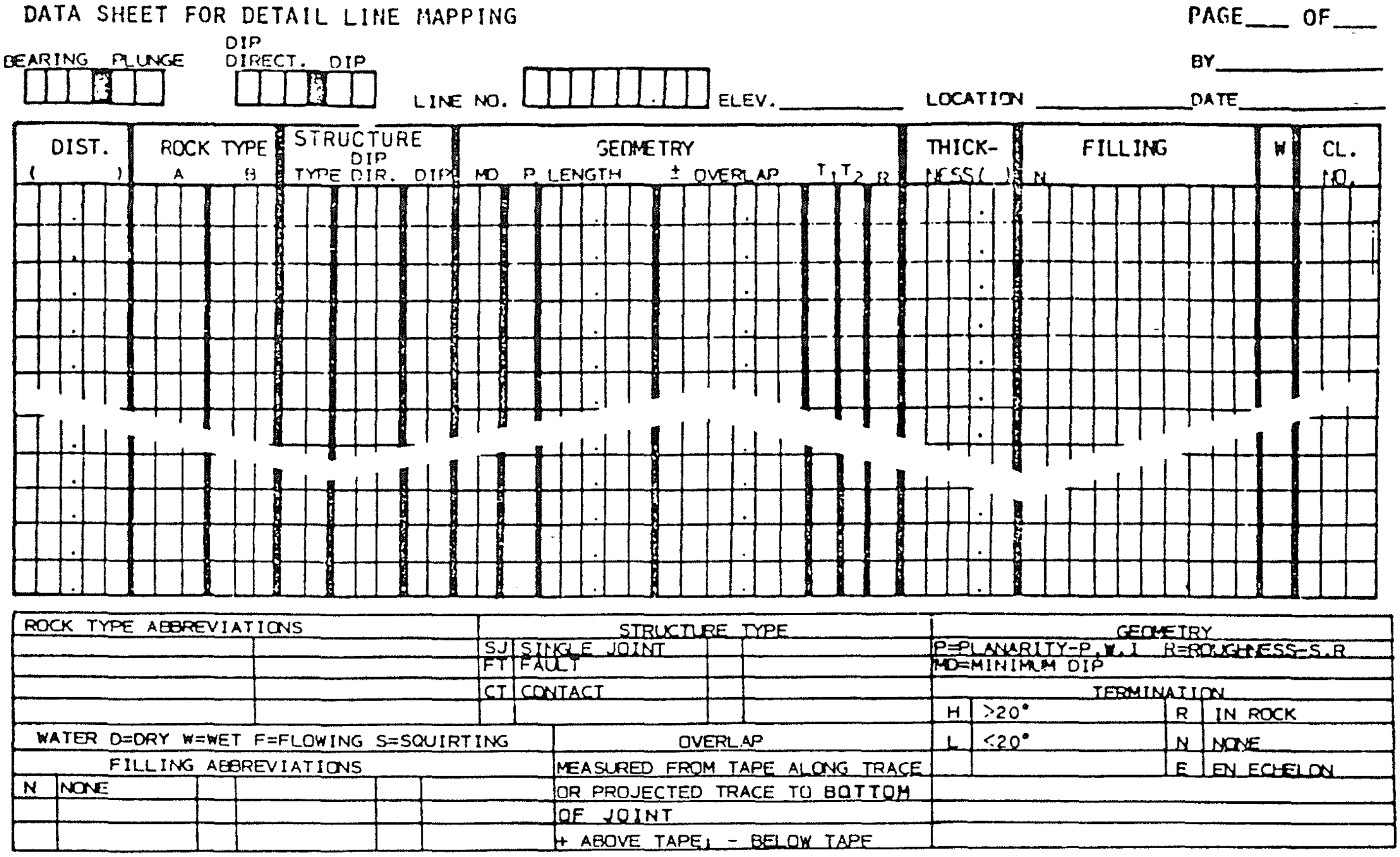
At the selected mapping site a tape is stretched along the rock exposure to intersect the greatest number of joints. The tape is used as a reference line. The mapping zone is parallel to the tape and commonly extends 1m above and 1m below the line. Depending on fracture intensity the width of the zone is generally decreased for heavily fractured rock and increased for more massive rock. Fractures falling within the mapping zone are described and the information is recorded on a tabular data sheet (Figure 2).

Only fractures with trace lengths greater than a cutoff length of 15 to 30 cm are measured since measurement errors increase with decreased length and detail below this cutoff is generally unsatisfactory.

Basic information at each site should include the bearing and plunge of the reference line, the dip direction and face angle of the rock exposure, the line number, the elevation, and location. A mnemonic system should be set up to identify rock types, fillings, and structure types.

Joint characteristics to be measured in the survey include:

1. Distance along the tape where the fractures or their projections intersect the tape. Joint set spacings are calculated from the intercept distances, the joint set orientation, and the line orientation. Spacings for joints with strikes parallel to the tape must be obtained from lines of different orientations or other backup investigative techniques, such as oriented core, but joints parallel to the line will have a distance measurement recorded where the orientation measurement is taken.



DATA SHEET FOR DETAIL LINE MAPPING

PAGE___ OF___

BEARING PLUNGE

DIP DIRECT. DIP

LINE NO.

ELEV.________

LOCATION __________

BY__________

DATE__________

DIST. ( )	ROCK TYPE A	ROCK TYPE B	STRUCTURE TYPE	STRUCTURE DIP DIR.	STRUCTURE DIP	GEOMETRY MD	GEOMETRY P	GEOMETRY LENGTH	GEOMETRY ± OVERLAP	GEOMETRY T_1	GEOMETRY T_2	GEOMETRY R	THICKNESS( )	FILLING N	W	CL. NO.

ROCK TYPE ABBREVIATIONS	STRUCTURE TYPE	GEOMETRY
	SJ SINGLE JOINT	P=PLANARITY-P,W,I R=ROUGHNESS-S,R
	FT FAULT	MD=MINIMUM DIP
	CT CONTACT	TERMINATION
		H >20° R IN ROCK
WATER D=DRY W=WET F=FLOWING S=SQUIRTING	OVERLAP	L <20° N NONE
FILLING ABBREVIATIONS	MEASURED FROM TAPE ALONG TRACE	E EN ECHELON
N NONE	OR PROJECTED TRACE TO BOTTOM	
	OF JOINT	
	+ ABOVE TAPE; - BELOW TAPE	

Figure 2. Detail Line Data Sheet

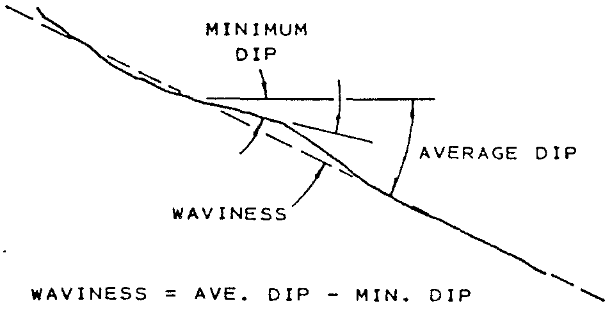


Figure 3. Minimum Dip Measurement to Determine Waviness

2. Rock types recorded in the previously designated mnemonic code. Usually geologic contacts or major structures are not crossed during the mapping because an effort is made to remain in the same structural domain and only one rock type is recorded.

3. Structure type, such as a single joint, fault, shear zone, bedding, foliation, and so forth.

4. Structure attitude recorded as dip direction and dip angle.

5. Minimum dip, or the dip of the flattest portion of the fracture surface (Figure 3). The average dip minus the minimum dip gives the waviness angle for the joint set. Patton (1966) has discussed the effects of waviness angle of the failure surface on shear strength.

6. Length measured as the maximum traceable distance that can be observed (Figure 1). It is assumed that the fracture throughout is at least as long as the measured trace.

7. Overlap or percentage that the trace length of one fracture overlaps the trace length of an adjacent fracture in the same set. The trace length of a fracture is measured from the bottom of the fracture to the reference line (Figure 4). If the bottom of the fracture falls above the line, the distance is positive; if the bottom of the fracture falls below the line, the distance is negative. If a fracture trace continues past the toe of the bench or outcrop, the minus distance is measured from the toe. The true overlap

is calculated from the measured trace length, the joint orientation, and the line orientation. Overlap cannot be measured for fractures parallel to the reference line.

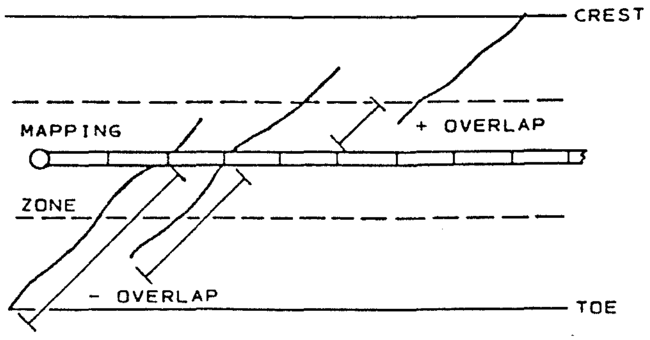


Figure 4. Measurement to Determine Overlap

8. Termination as observed at the top and bottom of the fracture in the dip direction. Termination types include unobservable termination for continuous joints, double termination where both ends of the joint are seen, single termination where only one end can be seen, in rock, and en echelon. Include notations on high- and low-angle terminations against other joints (Figure 5).

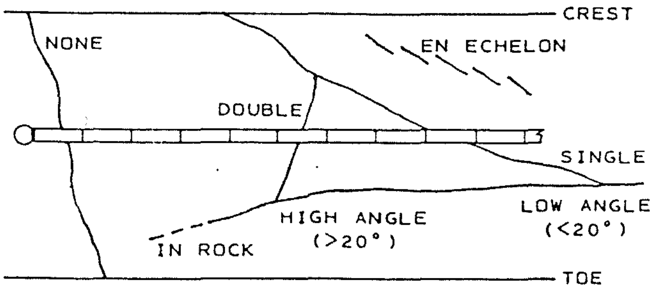


Figure 5. Types of Termination

9. Roughness, a qualitative measure of the small (2 cm or less) asperities on the fracture surface. Smooth and rough are the two categories used.

10. Thickness for filled joints, seams, and veinlets.

11. Filling types where fillings have appreciable thicknesses. Often more than one filling type is recorded and here a percentage of each type should be estimated and included as a comment on the back of the data sheet.

12. Water Conditions observable at the exposure.

Special Mapping Methods

After the detail line survey is completed and the rock fabric has been defined, special methods can be used to supplement the data on the set properties. The choice of the special mapping method depends on the size of the exposure and the nature of the fracturing. If the joint sets can be identified at the outcrop and the outcrop has sufficient size, either the recurrence interval method or fracture set mapping can be used.

The recurrence interval method was used by McMahon (1974) to obtain length distributions from fracture data plotted on geology plan maps. It is also possible to divide the exposure itself into increments (cells) and map the data for the fracture sets cell by cell. To determine the length distribution, the maximum length for each fracture set in each cell is measured, the number of cells containing each maximum length is determined, and the recurrence interval for the length is calculated as described by Gumbel (1954). Fracture length can be plotted versus recurrence interval to describe the extreme value distribution (Figure 6).

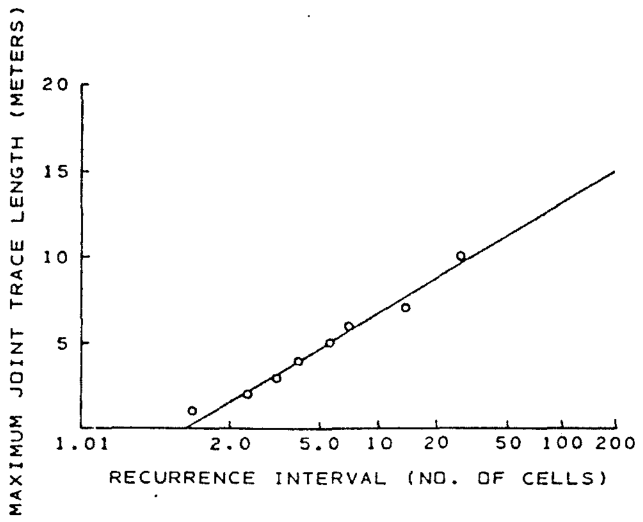


Figure 6. Extreme Value Joint Lengths

If this cell method is used, spacing and waviness angle can also be determined for the set by observing these properties for the joint set in each cell. From the mean value the coefficient describing the negative exponential curve is estimated.

In fracture set mapping, properties of the selected set are measured by procedures similar to those for detail line surveying until a sufficient number of data points have been taken to define the distribution. Again, 100 points per set is ideal, but generally the data only improve slightly as the number of observations increase past 30 for each set.

DETERMINATION OF DESIGN SETS

The first stage in data reduction is fabric analysis to choose the joint sets used in the design. Schmidt lower hemisphere point plots and contoured percent plots are constructed for each sampling site. In some studies, examination of the point plots readily reveals the preferred orientations. If the fracturing is very complex, statistical methods are available to analyze clusters of orientation data on a sphere (Mahtab and others, 1972). Statistical methods, however, are strictly numerical and do not include the engineering judgment that can choose design sets from close observation of the rock exposure. Engineering judgment can recognize more parameters than a statistical technique since the statistical analysis is rigid. Unless the statistical analysis is extremely complex, it has a limited number of variables to be considered, whereas a competent engineer who has mapped an outcrop can include geologic information that is difficult for a computer program to include. A more realistic approach in choosing design sets is to take advantage of the human ability to recognize spatial patterns and outline the design sets by inspection of the outcrop and the Schmidt plots. If statistics are used to combine sets the statistics should guide rather than control decision making.

Geologically, many fracture sets are present, but sets should be selected for design analysis based on their expected effect on slope stability. For this reason some sets, normally considered as individuals in the strict geological and statistical sense, are combined into one larger design set for analysis.

A Schmidt lower hemisphere point plot (Figure 7) and a contoured percent plot (Figure 8) are made for each detail line. Selection of design set limits is based on engineering judgment (Figure 9).

DISTRIBUTION OF DESIGN SET CHARACTERISTICS

Once the design sets have been determined, the various characteristics from all of the joints falling within the design set limits can be combined to form distributions. Each individual joint has dip, length, spacing, waviness angle, and overlap. Class intervals are chosen and histograms of each characteristic for each set are constructed. The curve that best fits the data is determined. By randomly sampling these curves

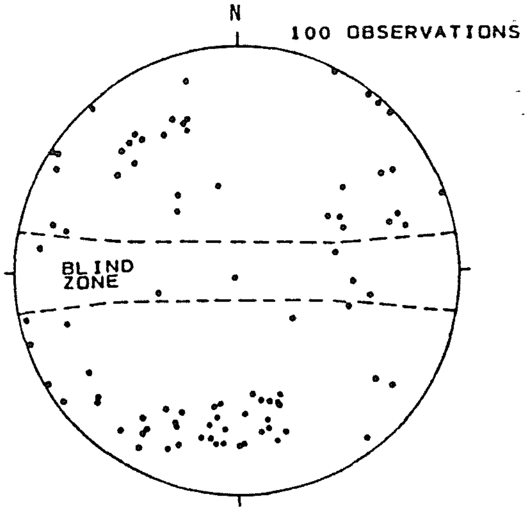


Figure 7. Point Plot

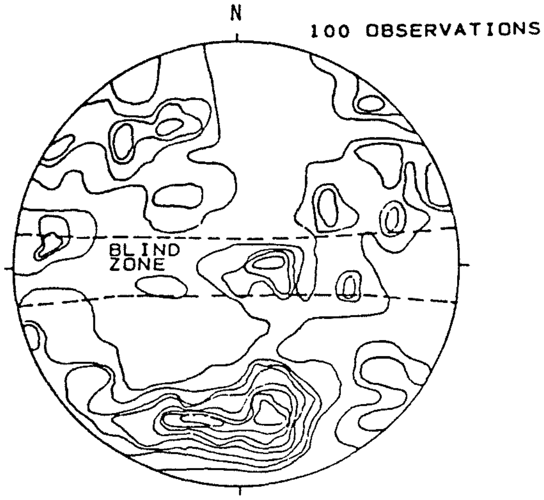


Figure 8. Percent Plot

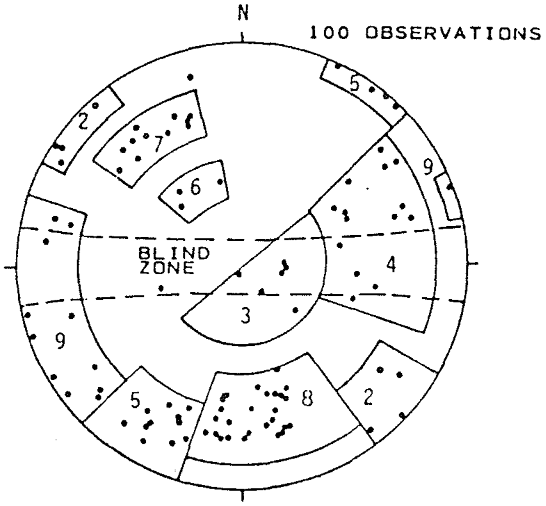


Figure 9. Design Sets

the stability analysis can be calculated iteratively to give a probability of failure.

Dip Distribution

Analyses of rock fabric from various open pits has shown dip distributions to be approximated by a normal distribution. All of the dips in the set are divided into classes, usually by five-degree intervals. The histogram is constructed based on the percentage of total number of observations in the set that fall into each class interval (Figure 10). A mean and standard deviation are calculated that can be used in the stability analysis or used to compute the probability of daylighting the design set with a specific slope angle. When the probability of daylighting is desired, the area under the normal curve can be calculated and the probability determined from standard statistical tables.

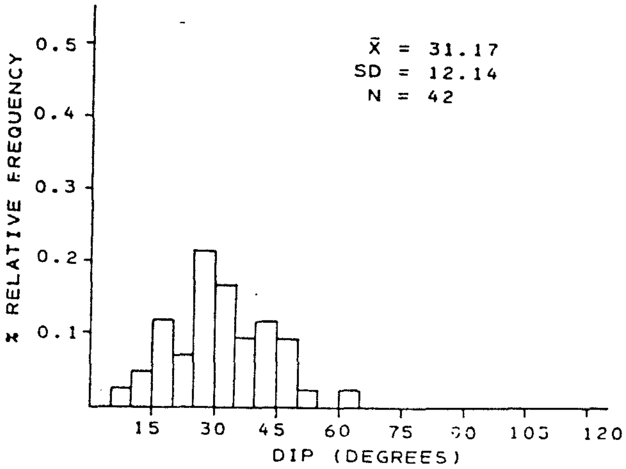


Figure 10. Typical Dip Distribution

Length, Spacing, and Waviness Angle Distributions

Length, spacing, and waviness angle of fracture sets basically have negative exponential distributions (Figures 11, 12, and 13). The measured characteristics are first accumulated in histograms, then a cumulative frequency distribution is fitted to the data forcing the curve through the data points and 100%. Then these distributions can be expressed in the exponential form, Y = exp-BX, to find the coefficient B describing the distribution. The coefficient B can be expressed also by the reciprocal of the mean value. Considering length, the probability P_L of having a fracture of a certain length L would be:

$$P_L = \exp -BL$$

If extreme value statistics are used the probability P_L considers the recurrence interval RI and the cell width CW.

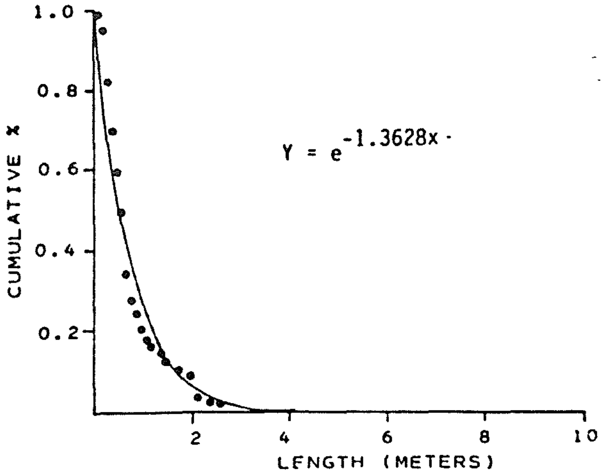


Figure 11. Typical Length Distribution

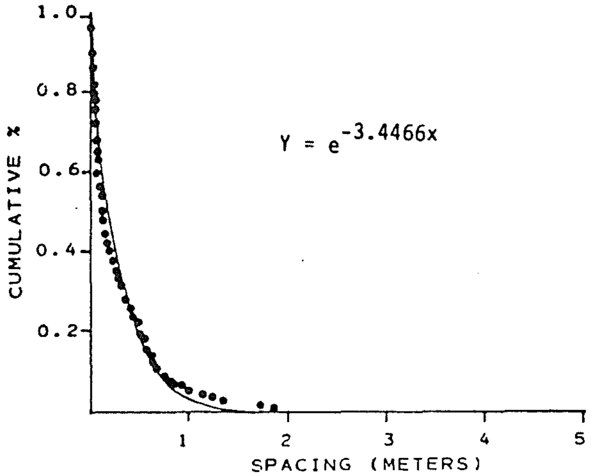


Figure 12. Typical Spacing Distribution

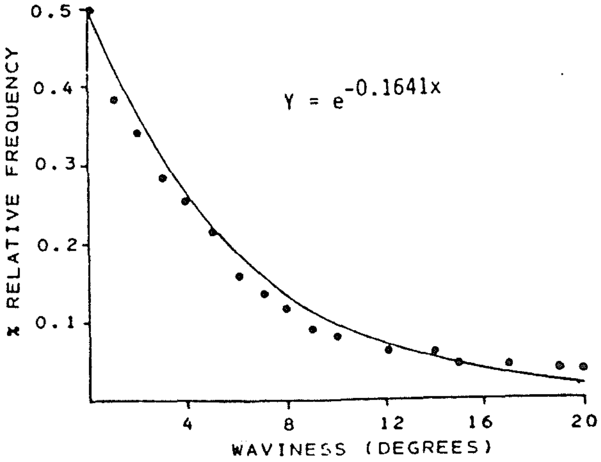


Figure 13. Typical Waviness Distribution

Spacing and waviness angle can also be described by the negative exponential distributions and their curves are determined using the reciprocal of the mean value for the coefficient B.

Robertson (1970) and Piteau (1973) have found that a negative exponential curve fits fracture length data.

On Figure 11 the length distribution shows an 0.08 probability of having a fracture of 2m or more, and nearly zero probability of having a fracture with a length of 4m or longer. At the same time the distribution curve shows that about half of the fractures in the set have lengths of 0.5m or longer.

Overlap Distribution

Jennings (1970) has considered the concept of overlap in the step-path failure mode. Ordered stepping has no overlap, random stepping 50 percent overlap, and semi-ordered stepping 25 percent overlap. Jennings believes the ordered stepping to be under-conservative, the random stepping overconservative, and the semiordered a reasonable intermediate value.

Overlap is still a relatively new measurement with considerable importance in the step-path geometry. The distribution common to this characteristic is uncertain but as more data becomes available from detailed joint surveys a known function may be found that fits the distribution.

Other Possible Distribution Forms

Although the normal and negative exponential curves seem to fit the data, other distribution forms are sometimes found. Often the more flexible Weibull distribution, of which the negative exponential is a special case, will result in a higher correlation coefficient. This distribution has the form, $Y = \exp -BX^C$, where C is yet another constant defining the shape of the curve.

McMahon (1974), has assumed a lognormal distribution for fracture lengths in conjunction with extreme value statistics to determine the probability P_L of having a fracture length long enough for failure. Triangular distribution or other forms can be used for some parameters in the stability models.

Basically, determining distributions is a curve fitting process to define a representation of the distributions to be used for Monte Carlo sampling. As long as the distribution is reasonable, resemblance to the data is probably good. If the distribution appears anomalous the data or the data processing system may be suspect, or it may be that anomalous distributions occur.

Difficulties Associated with Distributions

The determination of realistic distributions for the fracture properties is related directly to the method of sampling the fracture population. Of particular concern is the fracture length measurement. The observable trace length is measured and it is assumed the fracture throughout is at least the length of the measured trace length. If the fracture is doubly terminated a sample of known length has been obtained. If only one end is terminated the fracture is longer than the trace length measurement. Numerical methods to go from sampled population to target population using a truncated negative exponential distribution can be used to extrapolate for lengths.

A maximum and minimum truncation value must be determined. The minimum truncation value in the length distribution is the mapping cutoff (15 to 30 cm). The maximum truncation value is unknown and almost any distribution can be obtained depending on the assigned maximum truncation value.

The rock exposure represents a narrow "window" to observe the entire length population (Figure 14). If the upper truncation limit can be determined, better length distribution curves may result. Doubly terminated fractures lie within the "window", but if only these are considered, the amount of available data is severely restricted. A better approach would be to develop a mathematical correction to enable data from all fractures to be used.

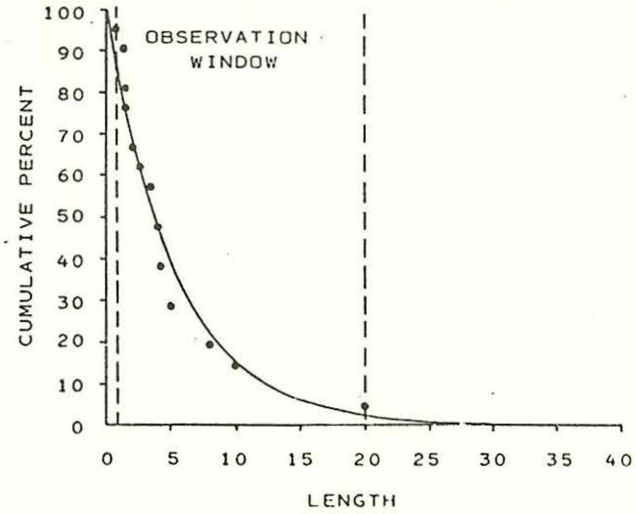


Figure 14. Observation "Window" for Fracture Lengths

Spacing can be calculated if the distance along the tape where the fractures or their projections intersect the tape is known. If the strike of the fracture set is within five degrees of the line bearing, an error in the spacing calculation can result. Instead, spacings for these sets that strike nearly parallel to the line must be obtained from a different detail line that is oriented more perpendicular to the strike of the fracture set. On occasion a fracture set may occur on the Schmidt plot from one line but be absent on a Schmidt plot from a second line in the same structural domain. This could result from either sample bias due to the line orientation with respect to the fracture set or the spacing of the set being greater than the line length.

Maximum values occur on the right end of a negative exponential curve and minimum values occur on the left end. Minimum lengths and spacings are more accurately defined than maximum lengths and spacings because maximum values are generally unmappable. For this reason spacing distributions may give reasonable values for the fracturing while length distributions may underestimate true fracture lengths.

Minimum dip is measured on a fracture surface to obtain the waviness angle. A fracture wavelength ranging between 0.5m to 5m is considered when minimum dip is measured on the detail line. Short wavelengths less than 0.5m are the small scale asperities. The effect of these asperities are accounted for in the results of large-scale or in-place direct shear tests. Wavelengths greater than 5m are dispersion in the dip of the design set. This dispersion is included in the dip distribution.

NON-DISTRIBUTABLE DESIGN SET CHARACTERISTICS

Some design set characteristics cannot be described by distributions but the effect of these characteristics on the design should still be considered.

Termination

The significance of termination in slope design can be conjectured.

One use of termination comes in determining fracture length distributions by considering only fractures that are doubly terminated. This aspect has been discussed previously.

A fracture terminating against another fracture at a high angle will have great difficulty in propagating further.

Fracture Filling, Thickness, and Water Conditions

Fracture filling and thickness can directly affect the shear strength of the fracture. If appreciable gouge is present the shear strength along the fracture may be the shear strength of the gouge.

Filling and thickness also influence permeability of the rock mass since large aperture openings and granular filling can produce a water carrying fracture. Water at the rock exposure may follow a preferred orientation that would be of major importance in the design of drainage systems to alleviate water pressures on the fracture surfaces.

USE OF DISTRIBUTIONS IN STABILITY ANALYSIS

Once distributions have been defined for the fracture characteristics, Monte Carlo sampling can be performed to develop potential failure geometries and strength parameters. Any input parameter that can be described by a distribution can be sampled and input to the stability equations. Safety factors can be calculated from each modeled condition. This procedure can be repeated until a distribution of safety factors is determined. Assuming that the safety factors fit a normal distribution, the probability of failure is represented by the area under the normal curve that has safety factors less than 1. If the safety factors are other than normally distributed, a better estimate of the probability of failure is the number of safety factors below 1 expressed as a percentage of the total number of iterations. This gives the probability of failure P_f along one continuous fracture.

The probability of failure P_f is only a portion of the stability equation for the entire slope where other probabilities must also be considered. Probability of instabilities for the entire slope can be used in a cost analysis as suggested by Call (1967, 1972) and McMahon (1971, 1974, 1975) for determining optimum slope angles and making management decisions.

CONCLUSIONS

Detail line surveys, recurrence interval methods, and fracture set mapping can be used to obtain information on fracture characteristics. Engineering judgment, Schmidt plots of the fracture orientations, and observations at the rock exposure are used to select the design sets. Each joint in the design set has length, spacing, waviness angle, and overlap. A distribution of these joint characteristics can be determined for each design set. A normal distribution seems to fit the dip. Negative exponential distributions seem to fit length, spacing, and waviness angle. The distribution to fit overlap is uncertain and requires more research. Other distribution forms may also fit the design set characteristics.

The length distribution of the target population is the most difficult to represent. The rock exposure is a narrow "window" to observe the entire length population. This means the length distribution may underestimate the true fracture lengths. It may be possible to devise a mathematical technique to adjust the length distribution and make it more representative.

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