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COMPOSITE PROBABILITY OF INSTABILITY
FOR OPTIMIZING PIT SLOPE DESIGN

PROBABILITÉ COMPOSITE DE L'INSTABILITÉ POUR
L'OPTIMISATION D'UN PLAN DE TALUS ROCHEAUX D'UNE MINE À CIEL OUVERT

KOMPOSITSPROBABILITÄT DER INSTABILITÄT FÜR
OPTIMIZIERUNG DES BÖSCHUNGENS IM TAGEBAU

COMPOSITE PROBABILITY OF INSTABILITY FOR OPTIMIZING PIT SLOPE DESIGN

by
Richard D. Call and Young C. Kim

ABSTRACT

The CANMET slope design manual presents an economic analysis for slope optimization that requires input of the probability of instability for a range of slope heights and slope angles. The expected mine life is divided into time periods and the slope geometry at the end of each time period is determined to obtain slope heights and slope angles. An appropriate probability of instability is sampled in a Monte Carlo simulation to assign potential slope failures to each time period.

In jointed rock, the increase in slope height during a mining period exposes a number of structures. Consequently, the probability of instability for a single structure at the toe of the slope is inadequate. Also, in most slopes more than one failure mode is possible, and a composite probability of instability must be determined for each slope height.

The effect of the number of joints exposed in each mining increment can be determined by a binomial expression. For 2-dimensional plane shear, the probability of instability, PI, for a mining increment is:

$$PI = 1 - (1 - P_F)^N$$

where P_F is the probability of instability for a single joint and N is the expected number of joints exposed in the interval. Other modes of failure such as step path and wedge failure can be considered in a similar manner.

If the same cost of failure can be assumed for different modes of failure, the composite probability of instability can be expressed as:

$$IP_F = 1 - \prod_{M=1}^{N} (1 - PI_M).$$

When the modes of failure have different costs, each mode can be sampled independently. If in simulation of the mining period more than one failure is predicted, the higher cost can be used.

INTRODUCTION

During the development of the methods of integrating slope stability with mine planning it became apparent that the probability of failure for a single failure surface daylighted at the toe of the final slope was inadequate, and that the following factors must be considered:

1. Geometry of the slope as a function of time— a pit slope excavated over a period of time, up to 20 years or more. Slope instability must be evaluated throughout the mine life, not just for the final slope. Since the economic analysis is on a time increment basis (usually 1 year), it is logical to express reliability of the slope as an incremental probability of instability; that is, the probability that an instability will occur as the pit is deepened by one increment. This increment would be the increase in depth during a time period. The smallest increment would be one bench height, as for practical purposes the blasting exposes a full bench instantaneously.

2. The number of potential failure surfaces in a mining increment— for any specific failure mode the probability of instability is a function of the number of potential failure surfaces in the increment.

3. Multiple modes of instability— several possible modes of failure may be present in a slope. If the different failure modes have the same cost of failure the probabilities can be composited. Otherwise, the probabilities for each failure mode must be sampled repeatedly.

BENEFIT-COST MODEL

The benefit-cost analysis is a Monte Carlo simulation of the sequence of mining in a pit. The life of the mine is divided into time periods. The usual increment is one year, although other time periods can be used. The normal mining and processing costs, and the benefit from sale of the product are determined for each time period. The pit is divided into design sectors based on the geology and geometry of the pit. A schedule of probability of instability as a function

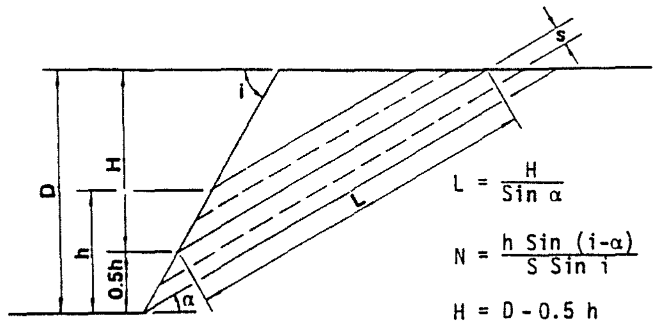

Figure 1. Simple Plane Shear Geometry

of slope height is determined for each sector.

In a simulation, the probability of instability for each sector in each time period is sampled. For those time periods where a failure occurs, the cost of the failure is added to the mining cost. The costs and benefits are then discounted to obtain a net present value. The simulation is repeated 30 or more times to get a distribution of possible outcomes. This analysis has been described in detail elsewhere (CANMET, 1976; Kim and others, 1976). The emphasis of this paper is on the development of the probability of instability schedule.

INCREMENTAL PROBABILITY OF INSTABILITY

The incremental probability of instability is the probability that a slope failure will occur when the pit is deepened by one mining increment, for example, one bench. This incremental probability is not the same as the probability that a failure will occur in any portion of the slope. This is an important difference. For example, the probability of instability schedule for a 38° slope in Table 1 shows a .002 probability of instability for a 1200 ft slope versus .039 for a 50 ft slope. This is an apparent contradiction to the usual concept that a higher slope is more unstable than a lower slope. What this means is that there is a lower probability of a continuous failure surface from the bottom to the top of a 1200 ft high slope than there is for a 50 ft high slope. The probability of having a failure at any point in the 1200 ft high slope with a mining increment of 50 ft is:

$$1 - [(1 - PI_{50})(1 - PI_{100})(1 - PI_{150}) \cdot \cdot \cdot \cdot \cdot (1 - PI_{1200})] \qquad (1)$$

For the 38° example in Table 1, this probability of instability is .257, which is higher than the .039 incremental probability for a 50 ft high slope.

The individual bench face angles would be about 55° for the 1200 ft high, 38° slope. The probability of having a bench failure somewhere in a vertical section of the slope is:

$$PI_B = 1 - (1 - 0.585)^N \qquad (2)$$

where N is the number of benches. When the slope is 400 ft high, N would be 8 and the probability (PI_B) is 0.999+; thus indicating at least one bench failure by the time the pit is 400 ft deep. The expected number of bench failures can be estimated by:

$$E(x) = N(PI_B) = 24(.585) \qquad (3)$$

which would be 14 bench failures for the 1200 ft high slope.

As can be seen in Table 2, the incremental probability of failure does not always decrease with slope height. For plane shear failure the primary factor is the length of the fracture along which failure could occur; a factor that results in a decrease in probability for an increase in slope height. Step path and rotational shear failure modes, on the other hand, have an effective continuous path and their incremental probabilities (PI) would increase with depth as a result of an increase in the stresses associated with higher slopes.

Table 1.
Probability of Instability Schedule for Sector I

Slope Height	Slope Angle 30°	38°	45°	55°
50 ft (15 m)	0.0	0.039	0.432	0.585
100 ft (30 m)	0.0	0.030	0.418	0.598
400 ft (122 m)	0.0	0.012	0.198	0.575
800 ft (244 m)	0.0	0.007	0.142	0.413
1200 ft (366 m)	0.0	0.002	0.086	0.251

Table 2.
Probability of Instability Schedule for Sectors II and III

Slope Height	Slope Angle 30°	38°	45°	55°
50 ft (15 m)	0.0	0.001	0.016	0.106
100 ft (30 m)	0.0	0.002	0.019	0.115
400 ft (122 m)	0.0	0.003	0.033	0.130
800 ft (244 m)	0.0	0.004	0.040	0.150
1200 ft (366 m)	0.0	0.005	0.047	0.170

Examples of incremental probability of instability schedules from Supplement 5-3, CANMET Pit Slope Manual

NUMBER OF FAILURE SURFACES IN A MINING INCREMENT

The effect on the probability of instability of the number of potential failure planes exposed as each mining increment is completed can be determined by a binomial expression. For the 2-dimensional plane shear analysis, the probability of instability (PI) for a mining increment is:

$$PI = 1 - (1 - P_F)^N \qquad (4)$$

where:

P_F = Probability of instability for a single fracture (Marek and Savely, 1978)
N = Expected number of fractures exposed in the mining increment

N for a plane shear analysis is calculated by (Figure 1):

$$N = \frac{h \text{ Sin } (i - \alpha)}{S \text{ Sin } i} \qquad (5)$$

where:

h = Height of mining increment
S = Fracture spacing
α = Mean spacing of fractures
i = Slope angle

Since failure could occur anywhere within the increment, the required length (L) for determining the probability of occurrence of a fracture length (P_L) is taken at the midpoint of the increment. This gives an effective slope height (H) of:

$$H = D - 0.5\,h \quad (6)$$

where D is the pit depth at the end of the mining increment (Figure 1).

Other modes of failure such as step path and simple 3-dimensional wedge failure can be considered in a manner similar to the plane shear geometry. For the step path, P_F is the probability of instability for a single step path, and N is the number of step paths that are within the mining increment. The number of step paths (N) is calculated from the spacing of the master joint set (Call and Nicholas, 1978). It is assumed that each master joint is part of a step path in the increment, although the probability of failure will depend on the percent of intact rock along the step path. Short continuous step path heights generally mean that a greater number of rock bridges are required for a failure of a given slope height, therefore, more breakage of intact rock would be required for instability.

Including intact rock bridges gives a length probability (P_L) for a given slope height of 1. The number of daylighted step paths within an increment, calculated according to equation (5), is based on the dip of the master joint.

In the simple 3-dimensional wedge analysis, N is the number of intersections within the mining increment. This N value is estimated from the 2 mean fracture set orientations, their mean spacings, and the slope face orientation. The length probability (P_L) becomes the probability of the trace length of the intersection being long enough to extend the full slope height.

COMPOSITE PROBABILITY FOR MULTIPLE FAILURE MODES

In most slopes more than one failure mode is possible. Also, it is possible to have more than one fracture set that could cause a plane shear failure. The probability of instability for the slope is found by combining the probabilities of instability (PI) determined for each postulated failure mode.

If the same cost of failure can be assumed for different failure modes then the composite probability of instability (IP_F) for the slope is expressed as:

$$IP_F = 1 - \prod_{M=1}^{N} (1 - PI_M) \quad (7)$$

where:

PI_M = Incremental probability of instability for a specific failure mode
N = Number of failure geometries present in the slope

For example, if 3 sets, a, b, and c, all form potential plane shear failure geometries in the same slope, and no other failure modes are considered, then the composite probability of instability for the slope is:

$$IP_F = 1 - [(1 - P_{Fa})(1 - P_{Fb})(1 - P_{Fc})]. \quad (8)$$

As a second example, if 2 fracture sets are considered to form a plane shear geometry (PS1, PS2) and there is a potential for step path (SP) and 3-dimensional wedge failure (W) in the same slope as well, the composite probability of instability is:

$$IP_F = 1 \quad [(1 - P_{FPS1})(1 - P_{FPS2})(1 - P_{FSP})(1 - P_{FW})] \quad (9)$$

In some mining situations different costs will be associated with particular failure modes, making it invalid to composite the probabilities for the benefit-cost analysis. In these situations the probability of instability for each failure mode is sampled independently in the benefit-cost model. If failures from more than 1 failure mode are predicted at the same time during simulation of the mining period, the highest cost failure is used.

CONCLUSIONS

Conventional stability analyses are not adequate for optimum pit slope design; however, these analyses can be used as part of a probabilistic analysis that considers the chance of failure on multiple failure planes.

To simulate the mining sequence it is necessary to calculate incremental probabilities of instability. Failure costs can be associated with these incremental probabilities and compared with the revenue derived from mining the ore to determine the optimum pit slope design.

In calculating incremental probabilities it is sometimes possible to have a decrease in the probability of instability with an increase in slope height. This results from analysis of jointing when the chance of a throughgoing fracture decreases as the required fracture length for failure increases, or when a failure mode becomes no longer viable for a high slope.

Often more than 1 failure can be postulated in the same slope at the same time. The correct incremental probability of instability is a composite of all of the probabilities from all failure modes that would cause instability in the slope.

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