John M. Marek, Mining Engineer, and James P. Savely, Geotechnical Engineer,
Pincock, Allen & Holt, Inc., Tucson, Arizona.

PROBABILISTIC ANALYSIS OF THE PLANE SHEAR FAILURE MODE

ANALYSE PROBABILISTIQUE DES RUPTURES DU MODE DE CISAILLEMENT

DIE WAHRSCHEINLICHKEITSANALYSE DER SCHERKLUFTFLÄCHEBRUCH

PROBABILISTIC ANALYSIS OF THE PLANE SHEAR FAILURE MODE

by
John M. Marek and James P. Savely

ABSTRACT

The probability of instability for 2-dimensional plane shear failure of a slope is a function of the shear strength, orientation, and length of the fracture forming the potential failure mode.

The probability that a fracture has a dip within a given range (P_D) and also a sufficient length in that orientation to reach from the toe to the top of the slope (P_L), can be calculated from dip and length distributions obtained from field mapping. Using a rigid block analysis, the probability of sliding can be calculated by Monte Carlo sampling of the shear strength and fracture roughness distributions to determine the distribution of safety factors. The area of the safety factor distribution less than 1 is the probability of sliding (P_S).

The probability of instability along a single, planar surface is the composite probability of dip, length, and sliding, summed over the range of possible daylighted dips:

$$P_F = \sum_{i=1}^{N} P_{S_i} P_{D_i} P_{L_i}, \text{ where } N = \text{the number of dip increments.}$$

This probability of failure is for single plane shear sliding and does not take into account the number of possible failure planes or other failure modes.

INTRODUCTION

Slope instability from rock sliding along a single failure plane can be analyzed to determine the probability of slope failure.

Variability in estimates of rock mass properties and rock strength measurements implies the probabilistic nature of geologic phenomenon. A design based on average values or on a single value does not account for this variability. A high safety factor might be calculated by using average values for the geologic parameters, but because of the variability shown by the distributions of these parameters, a high probability of failure may also be present (Höeg and Murarka, 1974).[1] In addition, it is difficult to incorporate the traditional safety factor calculation into an economic analysis. The probability of instability, however, can be used with an economic risk analysis to determine an economic optimum design that considers the cost impact of failure (Kim and others, 1976).[2]

GEOLOGIC STRUCTURE

The simple plane shear failure geometry is analyzed to determine the probability of failure (Figure 1). The geologic structure must occur in an orientation that makes this failure mode viable. Previous field mapping and subsequent data reduction provide the statistical distributions of joint set length, spacing, roughness, and dip (Figure 2) (Call and others, 1977).[3]

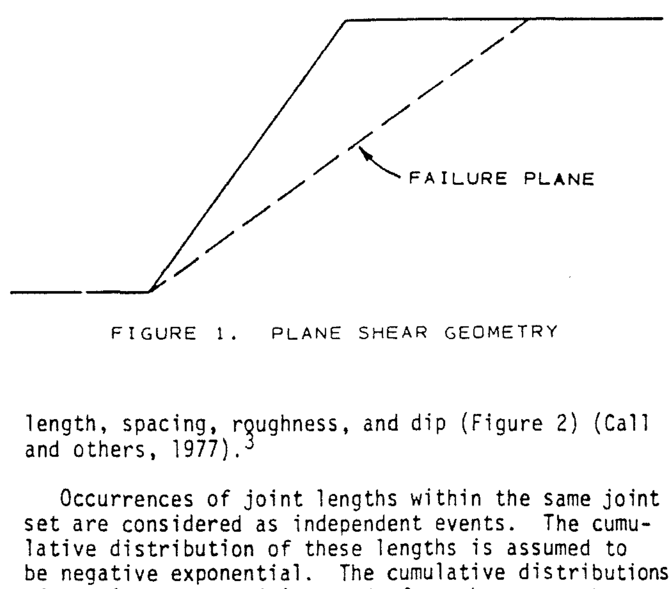

FIGURE 1. PLANE SHEAR GEOMETRY

Occurrences of joint lengths within the same joint set are considered as independent events. The cumulative distribution of these lengths is assumed to be negative exponential. The cumulative distributions of spacing between joints and of roughness on the joint surfaces are also assumed to be independent and negative exponential distributions. Joint dips within a given joint set are considered to be independent events and normally distributed. These statistical distributions of the joint set properties are used to model the occurrence of the failure plane.

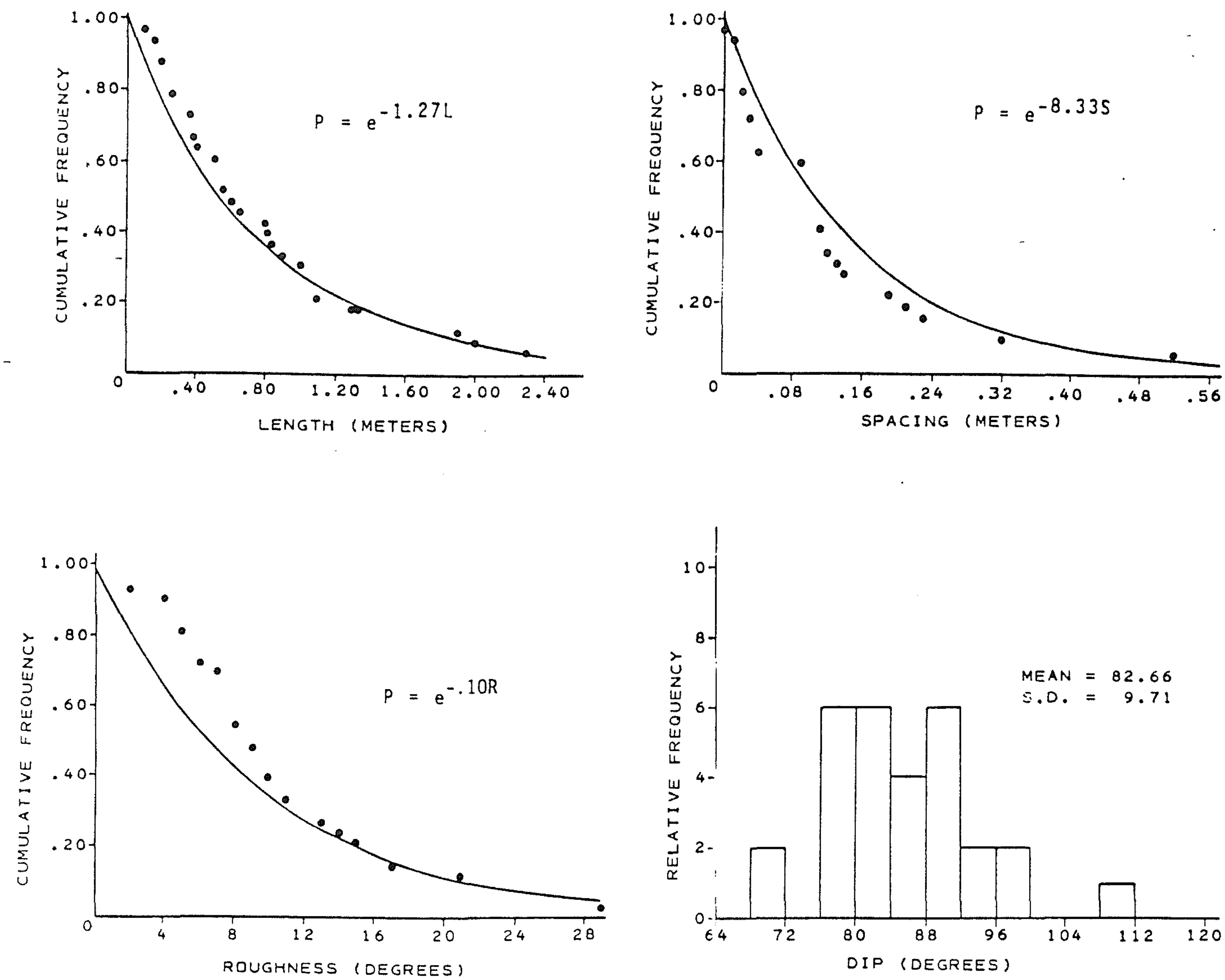

FIGURE 2. TYPICAL DISTRIBUTIONS OF JOINT SET CHARACTERISTICS

SHEAR STRENGTH

The required shear strength properties on the failure plane are determined by laboratory direct shear testing of joint surfaces. After primary reduction of test data to obtain a series of points representing shear strengths versus normal stresses, a curve is fit to the data. The Mohr-Coulomb straight line fit is most common:

$$\tau = c + \sigma_n \operatorname{Tan} \phi \qquad (1)$$

where:

τ = shear strength
σ_n = normal stress
c = cohesion
ϕ = friction angle

For samples tested under high normal stresses or when an extreme range of normal stresses is anticipated, the power failure criteria may be preferable:

$$\tau = K \sigma_n^M \qquad (2)$$

where:

τ = shear strength
σ_n = normal stress
K&M = curve parameters

This analysis uses the power failure law. The power curve is fit to the data by regression so as to minimize the squared error of the shear stresses over the range of the data. For any given normal stress, the mean expected shear strength at that normal stress is predicted by equation (2).

Testing uncertainties and natural variabilities in the fracture surface cause some dispersion of shear strengths about the mean for any given normal stress. This dispersion about the mean must be quantified to determine the probability of failure.

For a sample of a single joint, the variability of the mean shear strength can be expressed by calculating the relationship between the variance of the mean shear strength and the applied normal stress:

$$s^2[\bar{\tau}|\sigma_n] = s^2 \left[\frac{1}{N} + \frac{(\sigma o - \bar{\sigma}_n)^2}{(N-1)\ s^2\sigma_n}\right] \qquad (3)$$

where:

σ_o = applied normal stress
$\bar{\sigma}_n$ = mean normal stress
$s^2[\bar{\tau}|\sigma_n]$ = variance of mean shear strengths given a normal stress
s^2 = mean squared error of τ given σ_n (from regression)
N = number of points
$s^2\sigma_n$ = variance of the applied normal stresses

The standard deviation of shear strength for a given normal stress is equal to the square root of equation (3). The equation is used for only one joint sample; if several samples are considered, they are treated independently and the variation in mean shear strength is calculated from the mean power curve fit to each sample.

For example, if 6 samples of joints are tested, 6 power curves are generated, 1 for each sample. These 6 power curves are then combined into 1 new power curve to determine the new mean strength relationship.

The variation of the mean shear strength is determined by calculating a new standard deviation about the new mean curve. The standard deviation of shear strength varies with the normal stress, and can be approximated with a linear relationship. The form of the equation is:

$$s[\bar{\tau}|\sigma_n] = A\cdot\sigma_n + B \qquad (4)$$

where:

$s[\bar{\tau}|\sigma_n]$ = standard deviation of mean shear strength given a σ_n
n = normal stress
A&B = linear regression parameters fit as an approximation

Once the standard deviation of the mean is estimated, the shear strength distribution at a given applied normal stress can be defined. The applied normal stress is determined from the geometry of the slope and the failure plane. This normal stress is used in equations (5) and (6) to calculate the mean shear strength distribution:

$$\bar{\tau} = K\ \sigma_n^{\ M} \qquad (5)$$

$$s[\bar{\tau}|\sigma_n] = A\cdot\sigma_n + B \qquad (6)$$

where:

$\bar{\tau}$ = mean shear strength at σ_n
$s[\bar{\tau}|\sigma_n]$ = standard deviation of the mean shear strength given σ_n
K&M = curve parameters defining mean shear strengths
A&B = parameters of standard deviation curve

The specific details of fitting the power curve to the test data are presented in a paper entitled "Monte Carlo Simulation of Rotational Shear Analysis" by Paul J. Visca and John M. Marek.[4]

PROBABILITY OF FAILURE CALCULATION

The probability of plane shear failure is a combination of 2 probabilities: first, the probability that the failure plane exists (P_E), and second, the probability that sliding on the failure plane occurs (P_S). The equation for the probability of plane shear failure is:

$$PF = \int P_E\ P_S\ dx, \text{ integrated over all orientations } (x) \qquad (7)$$

This is the joint probability that a fracture is present and that sliding occurs, summed over all possible joint orientations. In the plane shear analysis, many orientations are not considered because their probabilities of sliding are zero. Only those joint sets with strikes that parallel the slope face and dips that could be daylighted are considered. In reality, the integral is approximated by:

$$PF = \sum_i P_{E_i}\ P_{S_i} \qquad (8)$$

where the summation in i is taken over a discrete set of dips with a range between zero and the slope face angle.

The summation of equation (8) is valid for any failure mode. For the plane shear condition, the probability of existence is the joint probability of 2 occurrences:

$$P_{E_i} = P_{D_i}\ P_{L_i} \qquad (9)$$

That is, the joint probability that the fracture has a certain dip attitude and that the fracture is long enough to reach from the toe to the top surface of the slope at that particular dip. The probability of length (P_L) is determined directly from the cumulative length distribution (Figure 2). For any dip in the joint set, a certain fracture length is required to reach from the toe to the top surface of the slope. The probability of meeting or exceeding this length is determined directly from the length distribution.

The probability of failure becomes:

$$PF = \sum_i P_{D_i}\ P_{L_i}\ P_{S_i} \qquad (10)$$

summed over a selected range of dips. This summed approximation to the integral, equation (8), is done in the following manner.

The slope face angle and the flattest dip to be considered are indicated on the normal distribution of fracture dips (Figure 3). The rock on rock friction angle minus 2 standard deviations can be used as a guide in selecting the flattest dip for input in the calculations. The area between the slope angle and the flattest dip is then divided into equal cells, generally in 2 degree increments (Figure 3). The probability of dip (P_{D_i}) is the probability that a dip falls within 1 of these cells. The probability of sliding (P_{S_i}) is calculated using the mid-point dip of the cell.

For example, within cell 5, P_{D_i} = .15 and P_{S_i} is calculated for a 35° dip angle. P_{L_i} is determined by calculating the length required for a fracture of 35° dip to reach from the toe to the top surface of the slope. Then the probability of meeting or exceeding that length is determined from the length distribution.

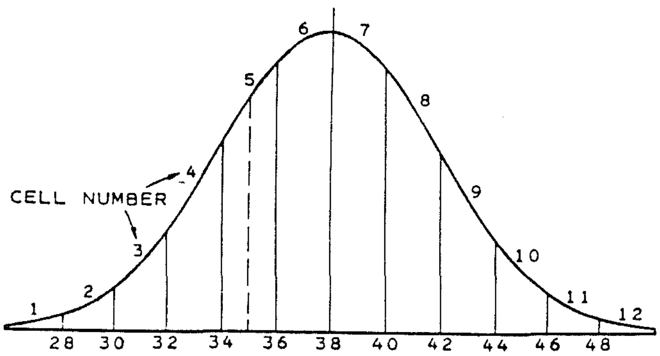

FIGURE 3. DIP DISTRIBUTION DIVIDED INTO CELLS

The probability of failure summation now becomes:

$$PF = \sum_{i=1}^{\#\ \text{Dip Cells}} P_{D_i} P_{L_i} P_{S_i} \qquad (11)$$

P_{D_i} and P_{L_i} were calculated from their distributions. Given that the fracture is present, the probability of sliding (P_{S_i}) is then calculated.

PROBABILITY OF SLIDING

Determination of the probability of sliding (P_{S_i}) uses Monte Carlo sampling. The geometry is determined by the slope face angle, the slope height, and the dip angle from the cell divisions (Figure 3). A different probability of sliding is calculated for the mid-point dip of each cell.

To analyze plane shear geometry, the normal stress and the driving stress are calculated on the failure plane. Any reduction of the effective normal stress due to pore pressure is considered at this time.

The actual Monte Carlo process now samples the mean shear strength and roughness distributions (usually 200 iterations). These 2 strength components are summed to determine a total resisting stress. The ratio of each resisting stress divided by the driving stress determines a factor of safety. The resulting statistical distribution of safety factors varies in form depending on the characteristics of the input data. The percentage of safety factors with values less than 1 equals the probability of sliding (P_{S_i}).

The variation in form of the safety factor distribution is primarily a function of roughness. If no surface roughness is present (that is, mean roughness angle = 0), the safety factor distribution is normally distributed because the shear strength distribution is also normally distributed. As more surface roughness is added to the shear surface (that is, mean roughness > 0), the resulting safety factor distribution becomes skewed to the right and no longer behaves as a normal distribution. The cumulative roughness distribution (Figure 2) is a highly skewed negative exponential distribution. It would appear that the safety factor distribution that results from the roughness and shear strength distributions would also be skewed to the right.

The safety factor distribution appears to be a Gamma distribution. For simplicity, a count is made of the number of safety factors with values less than 1. Dividing this count by the total number of Monte Carlo iterations gives a probability of sliding (P_{S_i}). Further work can be done to determine the precise distribution of safety factors.

It should be noted that the Monte Carlo process directly samples shear strengths rather than strength parameters such as K amd M from the power law, or friction angle (ϕ) and cohesion (c) from the linear relationship. This direct sampling from the shear strength distribution for a given normal stress is not affected by the dependence between curve parameters. For example, ϕ and c in the linear case have in the past been sampled as independent variables. In reality, there is a non-zero covariance between ϕ and c, and they are not independent. Sampling ϕ and c independently creates a wider dispersion in shear strengths than that measured in the laboratory. This wide shear strength dispersion causes a high dispersion in the safety factor distribution. This high dispersion will cause overestimation of the probability of sliding. By sampling shear strengths directly, the variability in the shear strengths of test samples is tied directly to the variability of mean shear strengths in the rock slope.

Once the probability of sliding is determined from Monte Carlo sampling, the probability of failure for the surface can be calculated using equation (11).

COMPOSITE PROBABILITY OF INSTABILITY

The final probability, PF, is the probability of failure for a single plane shear fracture. Where jointing is prevalent, there is a possibility of multiple failure planes. The next step would be to determine the probability that any one of these multiple planes will fail. Accounting for these planes requires the probability of failure, PF, and an estimate of the number of failure surfaces that are present in the slope. This number of failure surfaces can be estimated from the joint spacing distribution. The composite probability of instability must also include the possible occurrences of several different failure modes (Call and Kim, 1978).[5]

CONCLUSIONS

A probabilistic analysis of the plane shear failure mode can be performed if distributions of length and dip can be estimated. The probability of sliding can be computed from normal stress-shear stress curves and the failure geometry. Either the Mohr-Coulomb or the power failure criteria can be used to represent the shear strength.

The probability of failure for the surface can be calculated using equation (11). Unless the analysis is done for a known single fault plane or other major structure, the probability that considers multiple failure planes must still be determined. When plane shear failure along joint sets is considered, a composite probability of failure should be calculated from the estimated number of joint planes in the slope.

REFERENCES

1. Höeg, K. and Murarka, R.P., "Probabilistic analysis and design of a retaining wall", Journal of the Geotechnical Division, ASCE, GT3, March 1974, pp. 349-366.

2. Kim, Y.C., Cassun, W.C., and Hall, T.E., "Economic aralysis of pit slope design", paper presented at 11th Canadian Symp. on Rock Mehcanics, Oct. 1976.

3. Call, R.D., Savely, J.P., and Nicholas, D.E., "Estimation of joint set characteristics from surface mapping data", Ch. 9 in Monograph on Rock Mechanics Applications in Mining, ed. by W.S. Brown, S.H. Green, and W.A. Hustrulid, AIME, New York, 1977, pp. 65-73.

4. Visca, P.J. and Marek, J.M., "Monte Carlo simulation of rotational shear analyses", paper presented at 19th U.S. Symp. on Rock Mechanics, 1978.

5. Call, R.D. and Kim, Y.C., "Composite probability of instability for optimizing pit slope design", paper presented at 19th U.S. Symp. on Rock Mechanics, 1978.