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Evaluation of Material Properties

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ABSTRACT

Predicting the performance of a waste embankment requires an estimate of the properties of the waste and foundation materials. The performance characteristics and the associated material properties are:

Performance Characteristic	Material Properties
Stability	Shear Strength, Density
Settlement	Consolidation
Erodability	Grain Size, Weathering Index
Drainage	Permeability

Methods for obtaining estimates of the material properties include: laboratory and field testing, back analysis, and indirect estimates from other material properties. The most critical technical need is the development of indirect methods for use in feasibility studies and mine planning, prior to production when actual samples of waste material are not available.

Material properties are not unique, single values; they are probability distributions. There are two components to these distributions: the uncertainties in testing and the inherent variability of the material. The objectives of a material properties evaluation should be to obtain an unbiased estimate of the property distributions rather than the average or the minimum value. Whether the current trend to a probability design analysis is followed or a simple sensitivity analysis is conducted, the unbiased probability distribution is preferable to single values.

INTRODUCTION

Predicting the performance of a waste embankment requires an estimate of the properties of the waste and foundation material. The primary performance characteristics are shown in Table 1, with their direct material properties. Also shown in the table are indirect properties. Direct properties are considered here to be those properties which are input to mathematical solutions for predicting performance, such as shear strength used in a stability calculation. Indirect properties are those which are used to estimate direct properties from empirically- or experimentally-derived relationships.

In operating mines, estimates of material properties can be obtained by back analysis of existing dumps, field testing, and laboratory testing of samples, or indirect estimates from other material properties. In feasibility studies and mine planning prior to production, field testing can be conducted and samples taken of the foundation material; however, no waste material will be available in the final condition. The properties of the waste must, therefore, be obtained from simulated samples or indirectly estimated from in situ characteristics.

Sampling and Testing Strategies

Before discussing the specific material properties, some general comments on sampling and testing are in order. The success (or failure) of a materials testing program is more often determined by the design of the program (i.e., what tests are to be run and where and how many samples should be collected) than by the specific sampling and testing procedures. Although waste material is sometimes thought of as isotropic and homogeneous, an end dump waste embankment can be complexly variable, as shown in Fig. 1.

On the basis of geologic information and conceptual mine plans, the design analyses and required material properties should be established. Measuring everything about everything in the hope that out of this mass of data one can find what is wanted usually results in wasted time, effort, and money, and that vital parameter can still be missed.

If there is no information on which to base a design approach, the proper response is to divide

Table 1. Waste Embankment Performance Characteristics and Material Properties

Performance Characteristic	Direct Material Properties	Indirect Properties
Stability	Shear Strength Angle of Repose Unit Weight Stiffness Viscosity	Substance Compressive Strength Substance Shear Strength Specific Gravity Gradation Particle Shape Atterburg Limits
Settlement	Coefficient of Consolidation Unit Weight Void Ratio	Rock Type Mineralogy Soil Classification
Drainage	Transmissivity Storage Coefficient	Rock Type Mineralogy Soil Classification
Erosion	Grain Size Infiltration Capacity Clay Dispersivity Weathering Index	

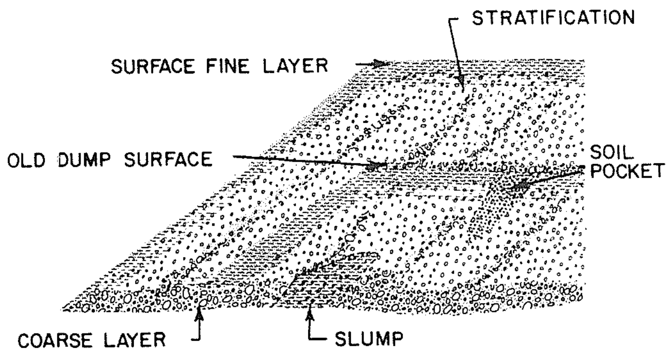


Fig. 1. Schematic cross section of an end dump waste embankment.

the material property investigation into two phases. The first phase would be surface mapping and a few exploratory drill holes to develop a geological model from which a rational design approach can be established and from which the second phase, detailed sampling and testing, can be planned.

Having a design model in mind does not mean that one should follow preconceived notions to the bitter end in spite of conflicting information. The principles of the observational method (Peck, 1969) are particularly applicable to the mine environment.

Material properties values resulting from a materials study are estimates of actual or potential properties, not absolute values. First of all, any tests or measurements are made of samples which may or may not be of the actual material in question. Take, for example, five direct shear tests of 0.09 m^2 (1 ft^2) samples of an embankment. They may not be from the shear plane along which the embankment will fail. Even if they are, assuming a 30 m by 30 m (100 ft by 100 ft) shear surface, they represent only 0.05% of the total shear surface. Secondly, the samples are probably not in the same state in the lab as they were in the field, and the stress conditions in the testing machine are not the same as actually exist on the shear surface in the dump. Thirdly, there is some element of measurement error in any testing equipment.

It is, therefore, not technically correct to say: "the shear strength of the embankment is ... ." The proper statement would be: "the shear strength of these samples, as measured in this equipment, under these consitions, is ... ," or "the estimated shear strength of the embankment, based on laboratory testing, is ... ." Since the addition of all the qualifers results in cumbersome verbiage, they are implied, if not stated, in subsequent parts of this paper.

Because of the inherent variability of the material and the uncertainty in sampling and testing, estimates of material properties are probability distributions rather than single values. (Statisticians would call them probability density functions.) The object of a material properties evaluation program should be to obtain an unbiased estimate of the probability distributions rather than simply the mean or minimum value. These distributions can be used in a probabilistic stability analysis to be input into a risk analysis. Economic optimization decisions on the tolerable level of risk should be made during the risk analysis, not when selecting shear strength for stability analysis.

Since it is not feasible to sample and test every point in a waste embankment or foundation, a simplified geologic model must be developed to plan the testing program. In order to develop this model, it is recommended that conventional geologic mapping and logging be supplemented by an engineering classification, such as the unified soil classification and the ISRM rock classification, during the site investigation. The combination of geologic description and engineering classifications is more effective than either one or the other for grouping similar materials and for determining continuity (or lack of continuity).

Ideally, sampling would be conducted after the model is developed; however, this would require redrilling. An alternative strategy is to sample extensively and to select the appropriate samples for testing from this library of samples.

A lithologic unit such as a stream deposit may be such a discontinuous interbedding of gravel, sand, silt, and clay that the actual spatial distribution of the material types within the unit cannot be modeled. In such a case, test results can be pooled, giving a wide dispersion of the estimated material properties for the unit. Alternatively, the test results can be divided into subsets of material types, and the spatial variability can be randomized based on the percentage distribution in the drill holes.

Shear Strength

Basically, the analytical methods of evaluating the stability of waste embankments use the ratio of the shearing resistance along a potential failure to the shear stresses along the surface as an estimate of stability. Although the methods of estimating stresses on a failure surface vary, shear strength is generally a direct input parameter.

The shear strength is a function of the normal stress. Coulomb, in 1776, demonstrated by the use of direct shear tests that this relationship can be approximated by a straight line, which leads to the classic shear strength expression

$$T = C + N \operatorname{Tan}(\phi),$$

where

T = shear strength,
C = cohesion,
N = normal stress, and
ϕ = friction angle.

On the basis of test results and theoretical considerations, granular material is often assumed to have a "zero" intercept (cohesionless). This simplifies the relationship to,

$$T = N \operatorname{Tan}(\phi).$$

From a design standpoint, this is a convenient relationship. At limiting equilibrium, where the shear forces equal the shear strength, the weight of

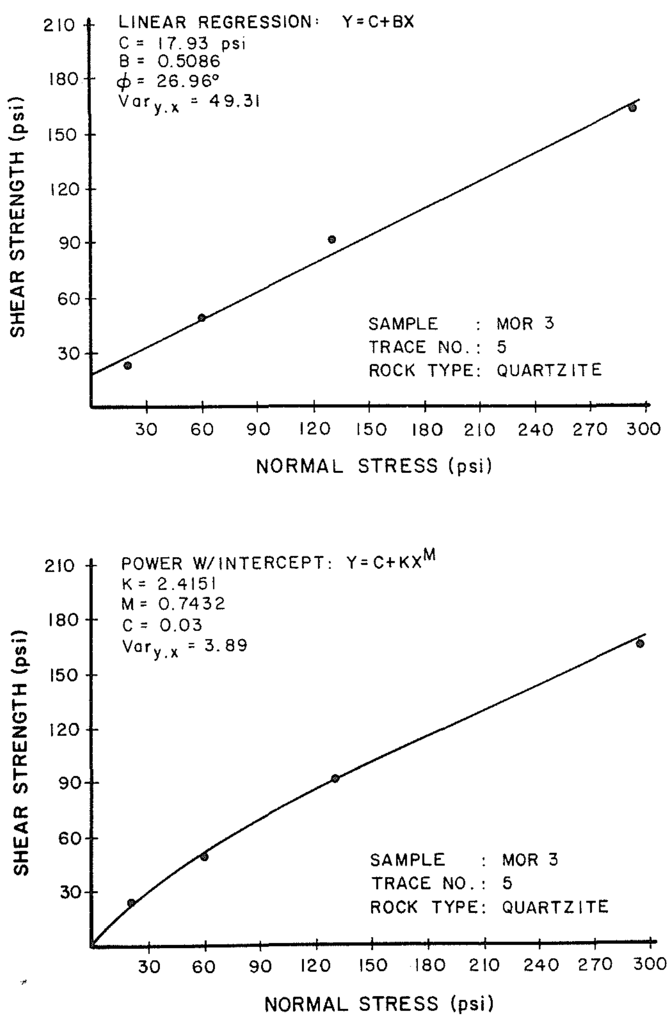
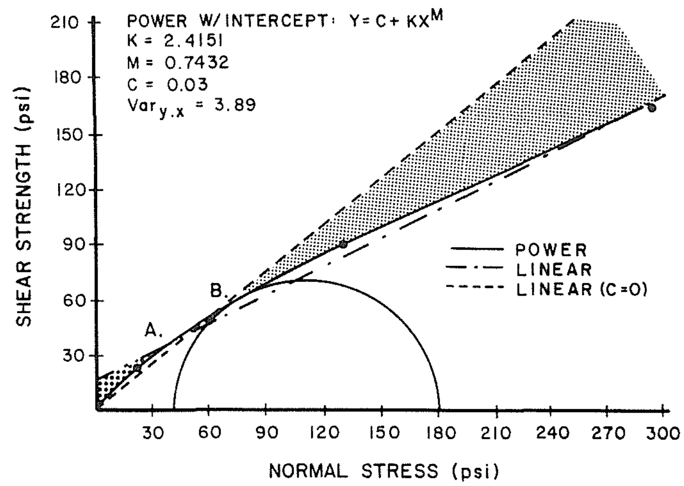
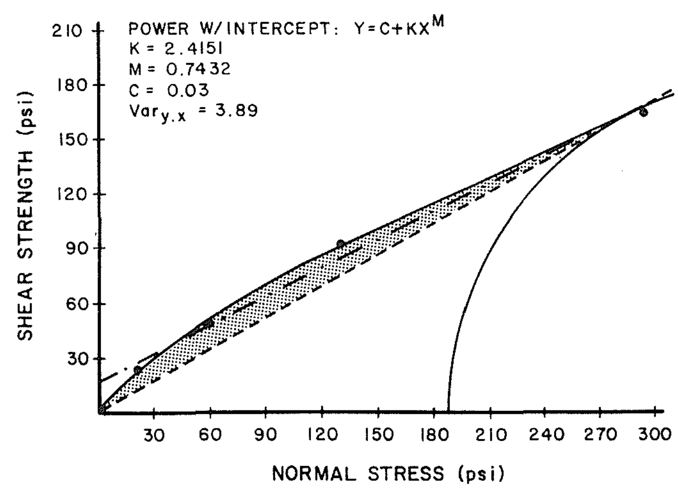


Fig. 2. Linear and power shear strength modes.



a) Overestimation at high normals.



b) Underestimation at low normals.

Fig. 3. Shear strength estimation error.

the material drops out, and the limiting equilibrium equals the friction angle. Choosing a slope angle less than the estimated friction angle is all that is required for a dry slope. Adding pore pressure where

$$t = (N - \mu)\ \mathrm{Tan}(\phi), \text{ and}$$

$$\mu = \text{pore pressure},$$

makes the design more complicated but still quite manageable.

There is considerable evidence that the shear-normal relationship is nonlinear, often with a cohesion intercept. The general power relationship

$$T = C + KN^M,$$

where K and M are constants, appears to be the best general fit. Both the linear and the cohesionless equations are special cases of the general power curve. (If M is 1, K becomes tanϕ). Power and linear curves for the same data are shown in Fig. 2.

The linear fit is a reasonable approximation in many materials, particularly over a limited range of normals. And, since some stability analyses are based on a linear strength model, there is an incentive to use it.

One objection to the linear fit with a cohesion intercept for waste is that granular material is cohesionless. This is hard to reconcile with the number of 15 m (50 ft) high slopes at 80° that I have seen cut in waste dumps. At Bingham, we had to drill and blast the old dumps to mine them.

The linear fit shown in Fig. 3a does overestimate the shear strength below Point A when compared with the power fit. So long as the anticipated normal stress is above Point A, it is not a problem. If the estimated normal stress is below Point A, a linear fit could be made to the power curve in the estimated range of normals.

An alternate practice of taking a line from 0 to the tangent of a Mohr circle from triaxial data to define a friction angle can lead to a significant overestimation of shear strength (area B, Fig. 3a).

This overestimation of shear strength at high normals is more serious than is the low normal overestimation of the linear with cohesion model because high normals would apply to large, deep-seated failures, whereas low normals would apply to small, shallow failures. If a Mohr circle with a high enough confinement is used, the overestimation is eliminated, but there is a consequent underestimation, as shown in Fig. 3b, which can result in a significant over-design.

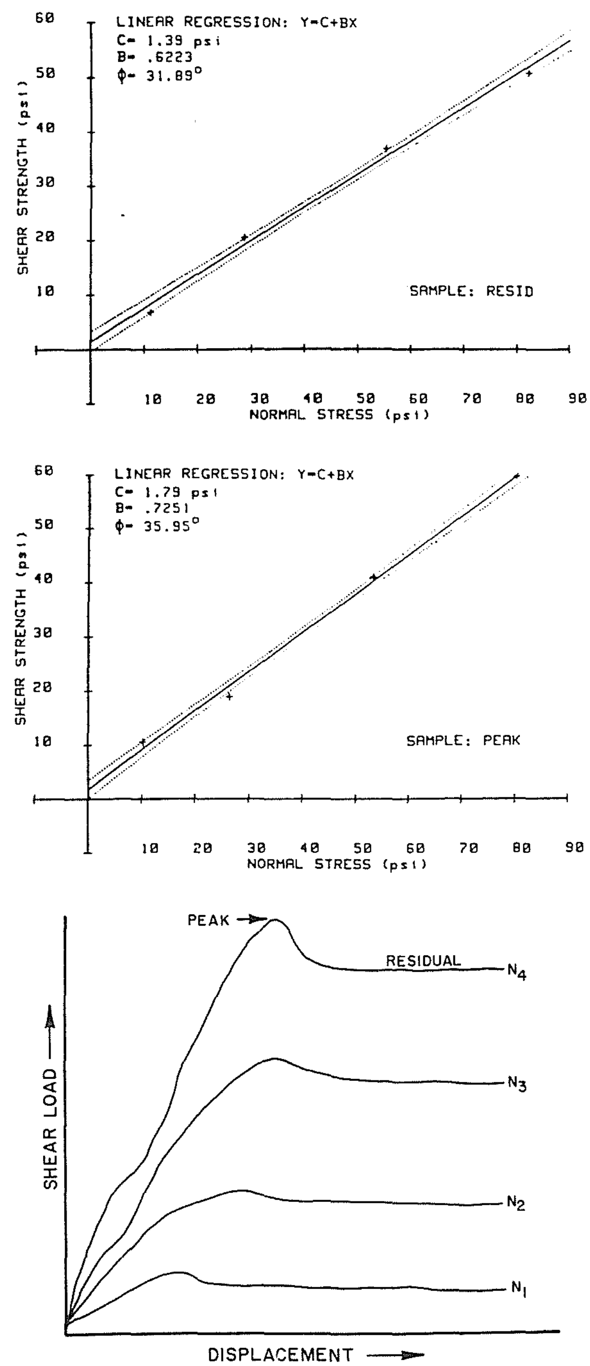


Fig. 4. Example of direct shear test results.

Methods of Testing

There is some controversy regarding the use of direct shear versus triaxial testing. My personal preference for slope design is direct shear, for the following reasons:

1) The direct shear test where a shear force is applied to a material under a specific normal is a better model of the forces acting on the bottom of a slice in a slope stability analysis than is the triaxial test.

2) The large strains required to reach residual strength cannot be conveniently attained during a standard triaxial test.

3) Direct shear is generally easier to run and is less expensive than triaxial testing.

Although cost may appear to be a mercenary reason for the choice of testing method, testing budgets are not infinite. Since the difference in shear strength between samples of a design unit is usually greater than the differences between testing methods, a number of inexpensive tests of a design unit will give a better estimation of the shear strength than will one expensive test. For the same reason, I do not usually advocate in situ tests.

Sample Size

For coarse rock dumps, sample size is a problem. Assuming that a sample is five times the particle size, dump material with 7.6 cm (3 in) cobbles would require a 4.6 m (15 ft) sample, admittedly not a particularly practical size. Since the shear strength is not as sensitive to absolute size as to particle strength, particle shape, and gradation, a scalped sample can give a reasonable representation of the material shear strength. Scalping at 6.4 cm (2.5 in) would be required for a 0.3 m (1 ft) shear box.

REDUCTION OF TEST DATA

During direct shear testing, a series of normal loads versus shear loads is recorded for each of several specimens, constituting one sample (Fig. 4). The applied stresses are calculated from these loads so that for each specimen the data points are plotted as a series of applied, normal stresses versus shear strengths.

For the Mohr-Coulomb model, a straight line is fit to these data points by linear regression. The equation of the line is given below. Assume,

$$\bar{\tau}_i | N = \hat{c}_i + N\hat{\mu}_i, \qquad (1)$$

where

$\bar{\tau}_i | N$ = mean shear strength, at a given N, for specimen i,
$\hat{c}_i$ = best estimator for the "cohesive" component of shear strength for specimen i,
$\hat{\mu}_i$ = best estimator for the "frictional" component of shear strength for specimen i (coeficient of friction), and
N = applied normal stress.

The calculation for $\hat{c}_i$ and $\hat{\mu}_i$ is based on the linear regression equations given below.

$$\hat{\mu}_i = \frac{\sum_{j=1}^{n} \tau_j \sigma_j - n\,\bar{\tau}\,\bar{N}}{\sum_{j=1}^{n} N_j^2 - n(\bar{N})^2} \qquad (2)$$

$$\hat{c}_i = \bar{\tau} - \bar{\sigma}\,\hat{\mu}_i ,$$

where

$\bar{N}$ = algebraic mean normal stress,
τ = shear stress,
$\bar{\tau}$ = algebraic mean shear stress,
N = applied normal stress, and
n = number of data points.

For a single rock sample or specimen, the variation of the mean shear strength at a given normal stress is expressed by the equation of the variance:

$$\mathrm{Var}(\bar{\tau}_i | N) = s^2 \left[\frac{1}{n} + \frac{(N - \bar{N})^2}{(n - 1)s_N^2}\right. \qquad (3)$$

where

$\mathrm{Var}(\bar{\tau}_i|N)$ = variance of the mean shear strength at a given normal stress (N),
s^2 = expected squared error, and
s_N^2 = best estimator of the variance of N.

For a single sample, the mean shear strength and variance about that mean for a given applied normal stress can be determined, using Eqs. 1, 2, and 3, for a given applied normal stress. The mean strength and one standard deviation above and below the estimate of the mean are plotted in Fig. 5.

For a group of samples from a design unit, the strength estimates of the samples are combined to determine the distribution of shear strengths within that design unit. Loosely stated, the mean shear strength curve of the design unit is defined by the mean of the various sample means. The detail of the method is as follows: for a number of samples (k), the mean shear strength at a given normal stress is the mean of the mean shear strengths.

$$\bar{\tau}_N = \frac{1}{k} \sum^{k} \bar{\tau} | N \qquad (4)$$

or by substitution, Eq. 4 becomes

$$\bar{\tau}_n = \frac{1}{k} \sum_{i=1}^{k} (\hat{c}_i + N\hat{\mu}_i), \text{ or}$$

$$= \frac{1}{k} \sum_{i=1}^{k} \hat{c}_i + \frac{N}{k} \sum_{i=1}^{k} \hat{\mu}_i , \qquad (5)$$

where

k = number of specimens,
$\bar{\tau}_N$ = mean shear strength, at a given N, for a rock type, and
$\tau_i|N$ = shear strength, at a given N, for specimen i.

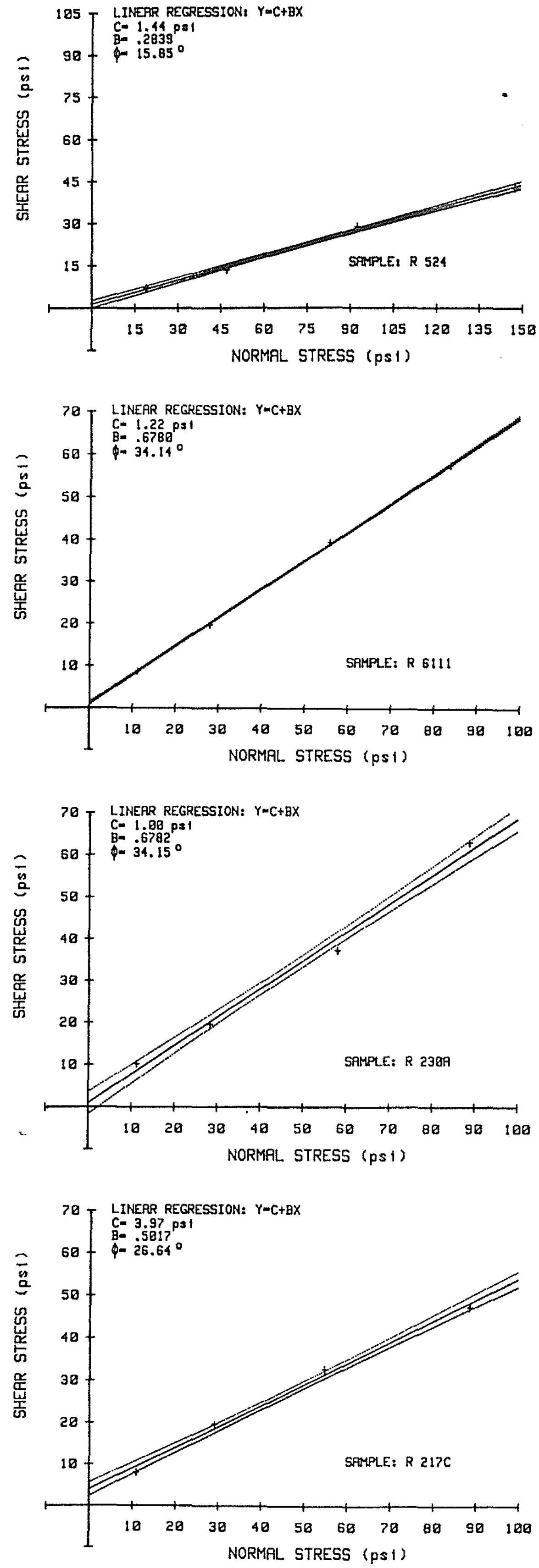
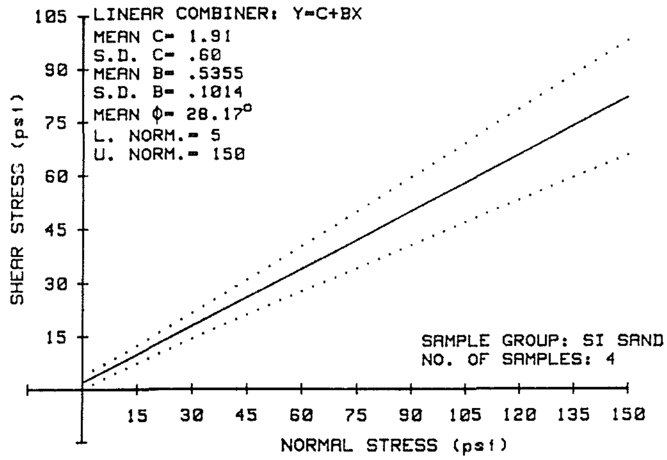
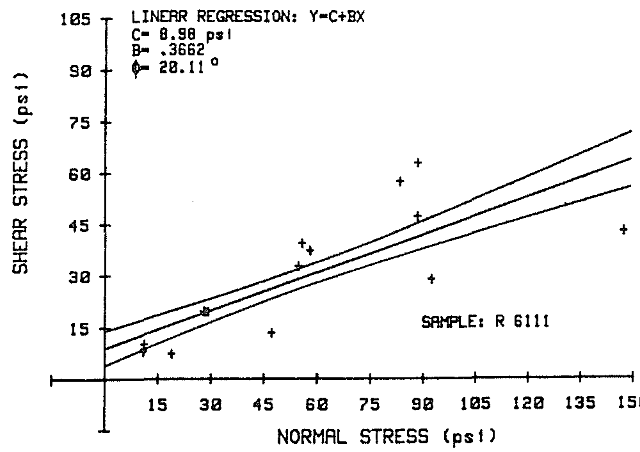


Fig. 5. Shear strength curve fits for a group of samples.



a) Combined sample curves



b) Regression fit to all data points

Fig. 6. Comparison of sample grouping methods.

The variance of the mean shear strength is based on the following equation:

$$Var(\overline{\tau}_N) = \frac{1}{k^2} \sum_{i=1}^{k} Var(\overline{\tau}_i|N) + Var(\overline{\overline{\tau}_i|N}), \qquad (6)$$

where

$Var(\overline{\tau_i|N})$ = variance of individual sample means.

The equation expresses the sum of the "within" sample variance and the "between" sample variance. Numerically, $Var(\overline{\tau}_N)$ becomes the mean of the variances plus the variance of the mean.

These equations are plotted in Fig. 6a, for the combination of the samples shown in Fig. 5. The solid line on Fig. 6a represents the mean shear strength for any particular normal stress on the horizontal axis. The two dotted lines are plus and minus one standard deviation of the mean shear strength. In other words, at any particular normal stress, the top curve is Eq. (5) plus the square root of Eq. (6). The straight line is Eq (5), and the bottom curve is Eq. (5) minus the square root of Eq. (6). So, for any particular applied normal stress, the mean and standard deviation of the mean shear strength can be defined.

The distribution of the mean is calculated rather than the distribution of the population on the basis that any slice in a stability analysis would be ten or more times as large as an individual test specimen. When a slice in effect contains ten or more "samples", the distribution of the mean is a better estimator of the slice strength than is the distribution of the population.

If all points from the individual specimens are plotted and a linear regression fit is performed as though they were all one sample, the resulting shear strength estimate would be different, as can be seen in Fig. 6b. Because of the mechanics of the regression method, the high normal point from sample 524 (Fig. 5) had a major impact on the fit. If one of the higher strength samples had been tested at the higher normal, the bias would have been reversed. The combining method described above avoids this problem.

Some may question the inclusion of sample 524 since it appears to be from a different population. These samples were selected from a larger number to illustrate the effect of fitting, which is present even with closer groupings. Also, as discussed earlier, it may be necessary to accept a large variability in a heterogeneous design unit.

A similar procedure can be used for the power model. The common method of making a logrithmic transform to a linear equation and using the linear regression minimizes the mean squared error of the log of shear and normal stresses and not the values themselves. Thus, a more sophisticated method should be used.

Triaxial Data

Fig. 7 shows three methods of fitting a shear equation to triaxial data. The conventional Mohr circle method is an empirical fit from which dispersion statistics cannot be obtained. The average stress versus deviator stress gives a deceptively good fit because σ_1 and σ_3 appear in both X and Y. Thus, the confining stress versus failure stress method is preferred.

The Mohr envelopes for large-scale triaxial testing of rockfill materials used by Marsal are shown in Fig. 8. Table 2 shows results of the linear fit to his data, as well as general properties of the material.

Indirect Estimates of Shear Strength

By analogy with his work on rock fracture strength, Barton (1981) has developed an indirect method of estimating rockfill shear strength. This estimation is of the form,

$$\phi' = R \log \left(\frac{S}{\sigma_n'}\right) + \phi_b,$$

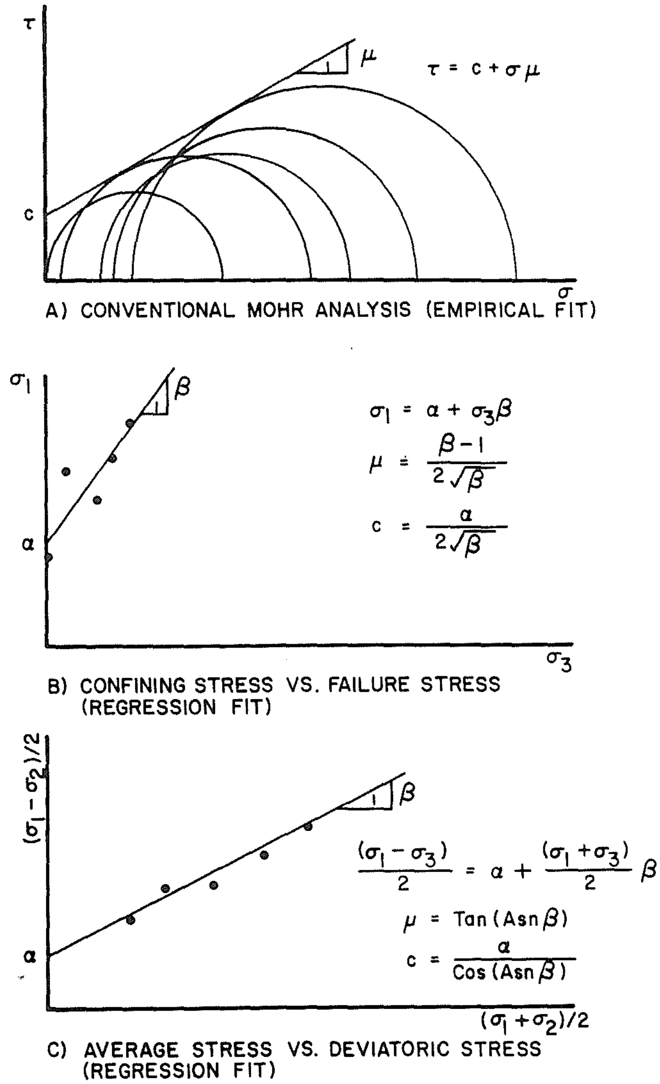


Fig. 7. Reduction methods for triaxial data.

where

ϕ' = the friction angle at normal stress σ'_n,

σ'_n = the normal stress,

ϕ_b = the basic friction angle of the particle material,

S = equivalent compressive strength, and

R = equivalent roughness.

The equivalent roughness is estimated for the particle shape and the porosity of the fill, as shown in Fig. 9a. The equivalent strength is estimated from the compressive strength (σ_c) of the rock substance and the particle size, as shown in Fig. 9b. This estimation technique produces a nonlinear shear strength curve similar to that derived from testing and is a promising alternative to testing.

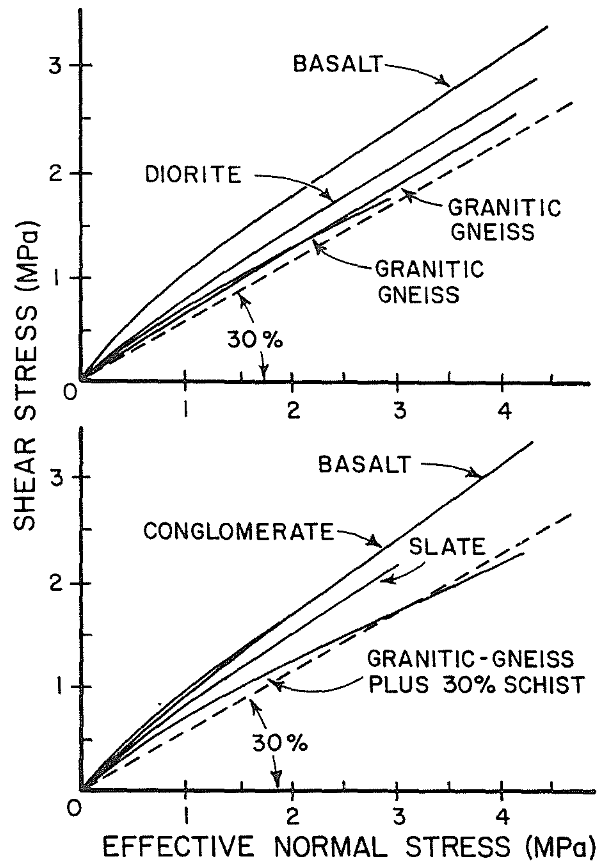


Fig. 8. Nonlinear Mohr envelopes for various rockfill materials (after Marsal).

Simulated Samples

During the feasibility stage and mine design, prior to mining, there is no waste material to test. If adits or shafts are driven to obtain bulk metallurgical samples, waste samples may be obtained for testing or measuring porosity, particle roundness, and gradation to be used in Barton's estimation.

Since the method of blasting would be different than that for an open pit and smaller haulage units would be used, effective particle size would tend to be smaller. This is not critical since shear strength is not very sensitive to particle size.

The uniaxial compressive strength and basic shear strength of the rock substance can be obtained from drill core.

For material with a low compressive strength (less than ± 1380 kPa (±200 psi), the particles will usually fail under the load in a waste embankment. For these materials, a sample can be simulated by mixing drill core or Shelby tube samples.

Angle of Repose

The height of an embankment and the particle size have little effect on the angle of repose. The

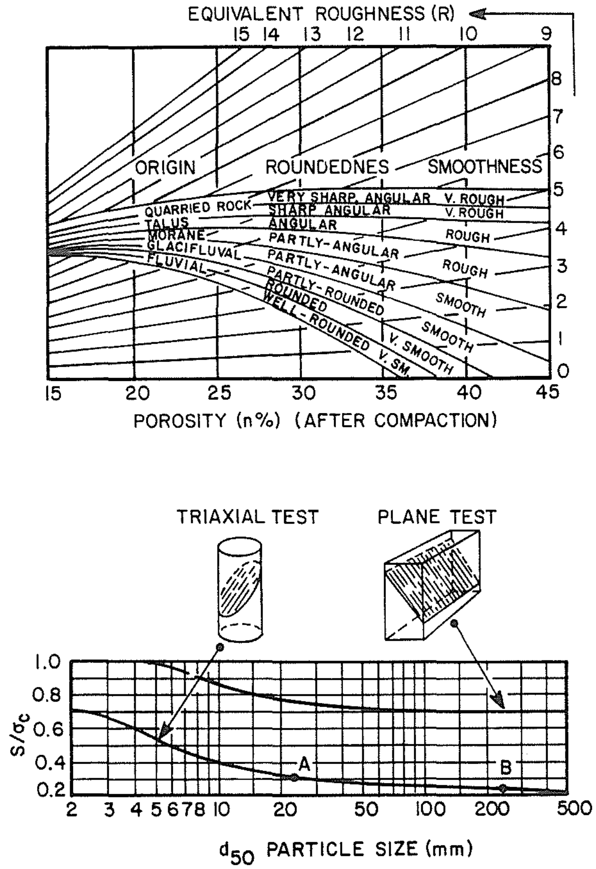


Fig. 9. Barton method of estimating rockfill shear strength (from Barton).

commonly observed angle of 37° has been measured for 183 m (600 ft) high dumps of mine run diorite and 15 cm (6 in) piles of 0.5 mm (0.02 in) quartz sand.

Particle shape and gradation and radius of curvature are the characteristics that affect the angle of repose. A well-graded material produces a smoother surface and more rounded particles roll more easily on a rounded surface. A concave slope will be a few degrees steeper than will a convex slope.

Observed angles range from 34° to 41° for common mine run materials. Flatter angles of old dumps may be misleading because settlement and erosion may have reduced the face angle.

Gradation

The size distribution of dump and foundation material can be obtained by screening. A sample size of one cubic yard may be required for coarse rock dumps. Because of segregation, the mine run rock may be a poor estimator of any specific location in the dump.

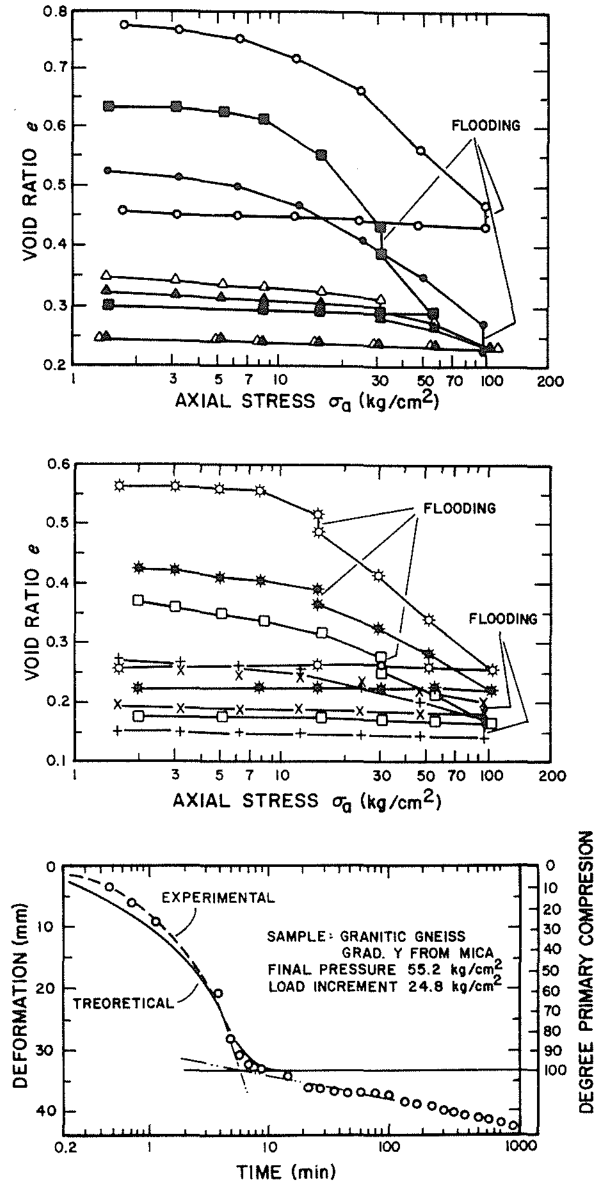


Fig. 10. Consolidation characteristics of rockfill (from Marsal, 1973).

Settlement

Settlement can be estimated by conventional consolidation tests. Marsal (1973) tested a variety of rockfill materials in a 114 cm (4.9 in) diameter oedometer. His results are shown in Fig. 10 and Table 3.

Table 2. Properties of Rockfill Materials (from Marsal, 1973)

SAMPLE	Origin	Shape	Effective Size d_{10}(mm)	Coefficient of Uniformity	Void Ratios e_i	e_d	e_l	Shear Strengths ϕ_m	C_m	Std. Dev. Parameters Tan ϕ	C
El Infiernillo silicified conglomerate	quarry blasted	angular	5.0	10.0	.45	.40	.55	34.68	17.08	.003	1.07
El Infiernillo diorite	quarry blasted	angular	20.0	5.0	.56	.48	.63	34.36	13.68	.001	.54
Pinzandaran sand and gravel	alluvial	rounded	0.2	100.0	.34	.29	.43	38.13	15.97	.002	.79
Malpaso conglomerate	quarry blasted	subangular	0.8	63.0	.38	.31	.51	36.52	17.42	.005	1.13
San Francisco basalt, grad.1	quarry blasted & crushed	angular	1.0	11.0	.35	.33	.55	35.95	32.50	.000	.22
San Francisco basalt,grad.2	quarry blasted & crushed	angular	1.1	18.0	.37	.29	.46	35.49	43.73	.001	.86
Mica Granitic-Gneiss grad.X	adit blasted	subangular	6.0	14.0	.32	.31	.50	30.96	16.66	.000	.08
Mica Granitic-Gneiss grad.Y	adit blasted	subangular	53.0	2.5	.62	.58	.77	30.68	9.84	.001	.70
Mica Granitic-Gneiss plus 30% schist grad.X	adit blasted	subangular	4.0	19.0	.32	.29	.51	31.75	9.02	.001	.35
El Granero slate, grad.B dense	quarry blasted	angular	27.0	4.3	.59	.64	.84	38.01	8.97	.003	.34
El Granero slate, grad.B loose	quarry blasted	angular	27.0	4.3	.69	.64	.84	32.65	10.73	.000	.02

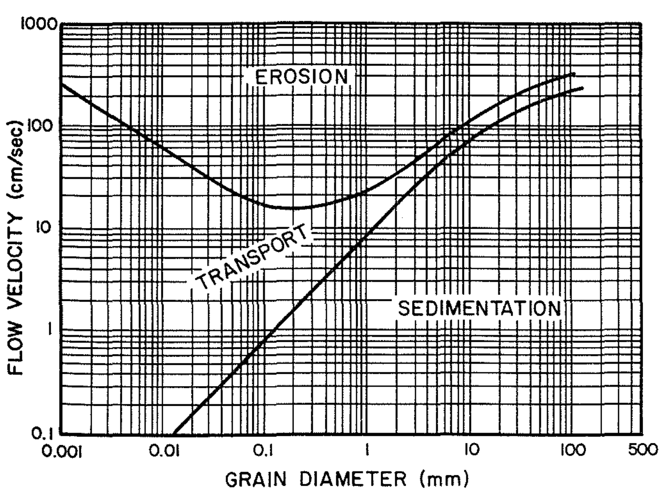


Fig. 11. Hjulström's critical drag curves.

Erodibility

The relationship between the critical water velocity at which erosion would occur and the grain size has been empirically established by Hjulström (1935). His curve, Fig. 11, shows a decrease in critical velocity with decrease in grain size down to a grain size of 0.2 mm (0.01 in); below 0.2 mm, the critical velocity increases with decreasing grain size. The explanation for the increase in critical velocity for fine-grained material is that the fine-grained materials have a higher cohesion and are, therefore, more resistant to erosion.

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Hjulström, F., 1935, Bull. Geol. Inst., UPPSALA 25, 221.

Table 3. Consolidation Characteristics of Rockfill (from Marsal, 1973)

Material	Sample No. and Symbol	Initial Void Ratio e	Coefficient of Compressibility, a_r (cm^2/kg) σ_a=2kg/cm^2	5	10	20	40	Grain Breakage B_g (%)	σ_a,max (kg/cm^2)
Sand and gravel from Pinzandaran	1 X	0.48	0.0041	0.0015	0.0010	0.0008	0.0012	7.8	101.2
Silicified Conglomerate from El Infiernillo	2 ○	0.80	0.0064	0.0051	0.0055	0.0045	0.0038	27.0	96.7
Diorite from El Infiernillo	3 ●	0.54	0.0058	0.0061	0.0053	0.0045	0.0025	28.3	96.9
Conglomerate from Malpaso	4 +	0.28	0.0035	0.0024	0.0022	0.0016	0.0009	11.9	96.8
Basalt from San Francisco, grad. 1	5 △	0.34	0.0040	0.0018	0.0009	0.0008	0.0010	1.3	105.5
Basalt from San Francisco, grad 2	6 ▲	0.32	0.0021	0.0013	0.0006	0.0005	0.0007	3.0	105.4
Granitic-gneiss from Mica, grad. X	7 □	0.37	0.0110	0.0045	0.0031	0.0025	0.0020	17.9	105.8
Granitic-gneiss from Mica, grad. Y	8 ■	0.63	0.0002	0.0033	0.0069	0.0081	0.0043	47.5	55.3
Slate from El Granero, grad. A	9	0.58	0.0073	0.0090	0.0072	0.0051	0.0027	25.5	100.8
Same	10 ✹	0.42	0.0065	0.0019	0.0025	0.0031	0.0020	19.1	101.2
Slate from El Granero, grad. B	11	0.75	0.0198	0.0111	0.0102	0.0060	0.0036	31.9	52.2
Same	12 ☼	0.56	0.0008	0.0017	0.0058	0.0051	0.0032	32.5	101.5

σ_a is the average axial pressure for each load increment.

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