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Initialization of non-uniform stress for complex geology and topographic conditions

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ABSTRACT: Typical mining problems involve complex geology with large topographic relief. These geologic and topographic features result in non-uniform stress field conditions. Valid performance of a numerical model requires an equilibrated modeled stress field that matches either measured or interpreted field stresses. Using traditional stress initialization methods, it is difficult to obtain a specific equilibrated non-uniform stress field in a complex numerical model. This paper will present a method to generate equilibrated non-uniform stress fields in a *FLAC3D* (Itasca 2006) model. The method explains how to modify component stresses in the *FLAC3D* model to match either measured or interpreted field stresses. Comparisons of pre-adjusted model stresses to post-adjusted stresses are provided.

1 INTRODUCTION

Large and complex three-dimensional numerical models are increasingly being used as tools in geotechnical engineering studies. Typical model grids feature severe topographic relief. In an effort to make the models as realistic as possible, numerous material types are included. The surface of the model grid may vary in elevations by over a kilometer or more. As *in situ* stress measurement data become more available, the need to incorporate this data into numerical models is rapidly increasing. Since confidence of forward simulated numerical solutions is predicated by a well calibrated stress model; a method to initialize verified field stresses is of paramount importance.

2 DEFINITIONS

The following notation will be used for this paper:
- Model orientation: (θ_z) is rotation in the XY plane (Fig. 1), relative to UTM or mine North; (X, Y, Z) is relative to the *FLAC3D* model coordinates

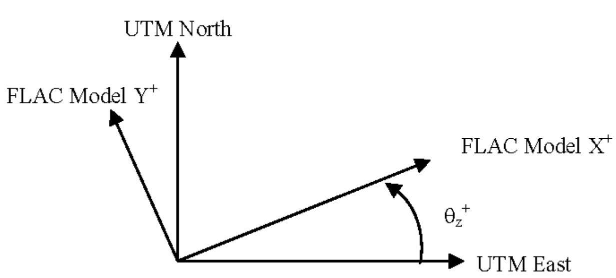

Figure 1. Rotation in the XY plane.

When used in formulas, (X) will indicate model direction East-West, (Y) will indicate model direction North-South, and (Z) will be elevation.

- Component stresses: $\{\sigma_{xx}, \sigma_{yy}, \sigma_{zz} \mid \tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \tau_{zy}\}$, where $\{\sigma$ = normal stress magnitude$\}$ and $\{\tau$ = shear stress magnitude$\}$.
- Principal stresses: $\{\sigma_1, \sigma_2, \sigma_3 \mid \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3\}$, where $\{\sigma$ = stress magnitude$\}$; $\{\alpha$ = azimuth of stress vector$\}$; and $\{\beta$ = dip of stress vector$\}$.

3 METHODOLOGY

There are five steps to the initialization of non-uniform stresses in the *FLAC3D* model grid. The steps are outlined below and then discussed in detail separately.

- Step 1: Calculate vertical stress as a function of depth below surface and zone specific gravity
- Step 2: Uniform stress initialization
- Step 3: Cycle grid to first non-elastic equilibrium state
- Step 4: Using vertical stress, adjust *FLAC3D* component stresses to match target stress
- Step 5: Cycle grid to final non-elastic equilibrium state and test goodness of fit

4 INITIALIZING THE VERTICAL STRESS

There are many ways to calculate a vertical pressure gradient on a static surface. One very effective way to do this in *FLAC3D* is through a series of user-defined FISH subroutines. First, the surface of the model, represented as a triangulated grid, is read into the *FLAC3D* model as array data. Next, for each zone in the model grid, a vertical (+Z) vector is projected from the zone center until it pierces the triangulated grid surface. The length of the vector provides the total depth. The incremental gravitational pressure contributed by each zone pierced by the vector is based on the vertical thickness and the specific gravity of the zone. The total vertical stress of the target zone is the sum of the incremental gravitational pressures added together. The calculated total vertical stress is then set in each zone in the grid using the INI command.

5 UNIFORM STRESS INITIALIZATION

Uniform horizontal stresses are now set in the model grid using the INI command. A simple method for determining the applied horizontal stresses is to assume a constant horizontal-to-vertical stress ratio. Horizontal stresses are then determined by simply multiplying the calculated vertical stresses by this ratio. For example, a horizontal-to-vertical ratio value of [1.5] would yield stresses in the horizontal directions one and one-half times the vertical stress.

$$\sigma_{xx} = \sigma_{yy} = 1.5\sigma_{zz} \tag{1}$$

Similarly, a pure gravitational only confinement could be initialized by a FISH subroutine that checks each zone's elastic properties and initializes the stresses based on the Poisson effect (Brady & Brown 1994).

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} * [\nu / (1- \nu)] \tag{2}$$

At this stage, all shear stresses are disregarded and set equal to zero. It is usually a good idea at this time to set up a few different run files at various confinements so that a set of save files can be generated in the next section.

6 INITIAL NON-ELASTIC EQUILIBRIUM

Initial equilibrium is now simulated in the model grid. Boundary conditions are standard, with roller sides and pinned corners. The top surface is free. It is good practice to obtain an elastic equilibrium solution before advancing to a non-elastic solution. This is to allow the elastic model stresses to adjust for severe topographic relief, such as is found in deep valleys or high peaks. In the case of gentle topography or a flat surface, usually only one *FLAC3D* time-step is needed to achieve an elastic equilibrium solution.

Once the elastic solution is obtained, non-elastic strengths are input to the model and the model is cycled to equilibrium. At this time, save files should be created for each of the horizontal confinements selected in the previous section.

7 MATCHING FIELD STRESS DATA

First, the principal stresses, either interpreted or from stress measurements, have to be rotated into the *FLAC3D* grid coordinate space. This is a direct rotational transformation. If the model grid is designed using the same coordinate space as the principal stress field to be simulated ($\theta_z = 0$), then no rotation is required. Otherwise the rotation is carried out by the matrix operation (Goodman 1980),

$$(\sigma)_{x'y'z'} = (L) * (\sigma)_{xyz} * (L^T), \text{ where} \tag{3}$$

$$(\sigma)_{x'y'z'} = \begin{bmatrix} \sigma_x' & \tau_{xy}' & \tau_{xz}' \\ \tau_{yx}' & \sigma_y' & \tau_{yz}' \\ \tau_{zx}' & \tau_{zy}' & \sigma_z' \end{bmatrix} \tag{4}$$

$$L = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$(\sigma)_{xyz} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{6}$$

$$L^T = \begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Once the resultant matrix $(\sigma)_{x'y'z'}$ is obtained, the target value of σ_z' is compared to the model σ_{zz} stress for each of the zones in the *FLAC3D* model grid and a ratio is obtained.

$$K_r = \frac{\sigma_{zz}}{\sigma_z'} \tag{8}$$

The ratio K_r is used to adjust the values of the other five components of stress to input for each zone in the model grid:

$$\sigma_{xx} = (\sigma_x')K_r \tag{9}$$

$$\sigma_{yy} = (\sigma_y')K_r \tag{10}$$

$$\tau_{xy} = (\tau_{xy}')K_r \tag{11}$$

$$\tau_{xz} = (\tau_{xz}')K_r \tag{12}$$

$$\tau_{yz} = (\tau_{yz}')K_r \tag{13}$$

With σ_{zz} unchanged, the other five component stresses are adjusted in the model grid with the INI command. Each material type in the model can have a different stress state initialized in their respective zones, and each zone will vary relative to the vertical stress it holds.

8 FINAL NON-ELASTIC EQUILIBRIUM

At this time, the model stresses are a direct match to the interpreted field stresses. The model must be cycled again to an equilibrium condition and the values cross-checked against the desired stress field. Numerous model runs may be necessary to obtain the best fit to the interpreted stress field data. There may be times when certain material types require adjustment and others will not. The final stress state used should be the one that best matches the interpreted field stresses.

9 EXAMPLE

In this hypothetical example, we have a severely varying topographic surface in an area of a copper porphyry deposit with three basic material types: a host sedimentary unit (limestone), an intrusive core (diorite), and a high clay-altered rock surrounding a fault contact zone (clay). A tunnel is to be driven through the faulted contact and we want to simulate stress changes during mining of the tunnel.

Assume that stress measurements in the sediments indicate that the major principal stress is flat or horizontal, and stress measurements in the diorite indicate closer to hydrostatic values. The challenge is to get the numerical model to reflect both the horizontal stress field in the sediments and, at the same time, maintain a more hydrostatic stress state in the intrusive material. For simplicity, assume the *FLAC3D* model is in the same coordinate space as the UTM coordinates, so no rotation transformations have to be applied to the stress measurement data. The principal stress fields to be simulated are:

- DIORITE @ [-235 m, 115 m, 5800 m]
  - $\sigma_1 = 21.5$ MPa $\alpha_1 = 285°$ $\beta_1 = 15.0°$
  - $\sigma_2 = 18.5$ MPa $\alpha_1 = 190°$ $\beta_1 = 20.0°$
  - $\sigma_3 = 18.0$ MPa $\alpha_1 = 50°$ $\beta_1 = 65.0°$
- LIMESTONE @ [185 m, 85 m, 5800 m]
  - $\sigma_1 = 24.9$ MPa $\alpha_1 = 109°$ $\beta_1 = 19.3°$
  - $\sigma_2 = 16.5$ MPa $\alpha_1 = 351°$ $\beta_1 = 52.5°$
  - $\sigma_3 = 14.2$ MPa $\alpha_1 = 211°$ $\beta_1 = 31.0°$

The model properties used in this example are given in Table 1.

Table 1. Model properties used in this example.

Material	Density-ρ (kg/m^3)	Young's Modulus-E_m (GPa)	Poisson's Ratio-ν	Phi-ϕ (degrees)	Cohesion-c (MPa)	Tensile Cut-t (MPa)
Diorite	2650	35.0	0.25	40.0	2.05	0.90
Limestone	2800	25.0	0.25	38.0	1.40	0.60
Clay	2450	7.0	0.35	25.0	0.40	0.15

First, the vertical stresses are calculated from a triangulated grid and then a uniform hydrostatic horizontal stress field is initialized. Figure 2 shows the initialized vertical stress contours for the model grid.

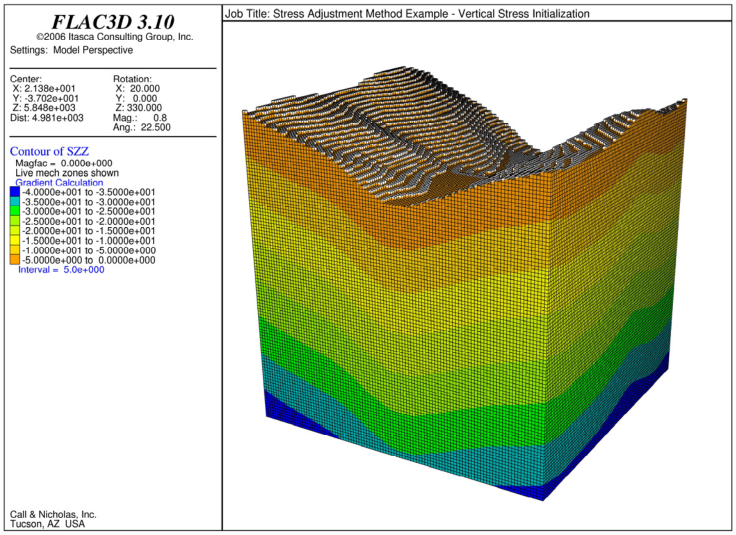

Figure 2. Initialized uniform stresses in the *FLAC3D* model.

The model is cycled to an elastic and then a non-elastic equilibrium condition. A number of zones near the measurement locations are sampled to obtain a representative average stress field. The modeled stress field statistics are represented as lower hemisphere equal area Schmidt plots showing the measured stress and the sampled modeled stresses. The goodness of fit is determined by using a method developed by Call & Nicholas, Inc., called a "sum-difference-vector" (SDV) calculation. A difference vector is defined in three dimensions as the vector connecting the tips of two independent vectors. The magnitude, azimuth, and dip of the principal stress vectors contribute to the length of the difference vector. The sum of the difference vectors is the total error of the modeled stress relative to the measured stress. An error of zero means that there is an exact match. Figures 3 & 4 show the pre-adjusted modeled stress fields and SDV.

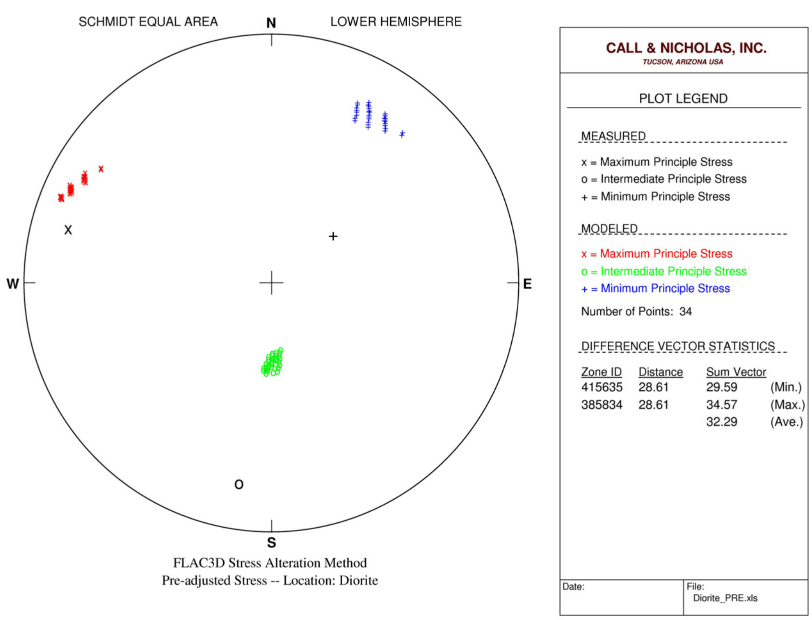

Figure 3. Initial stress condition (pre-adjusted) for the Diorite material.

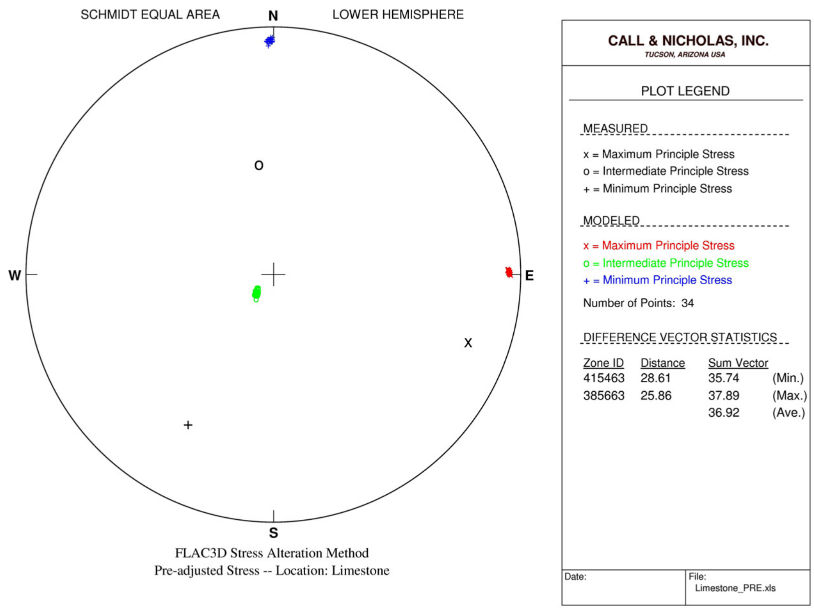

Figure 4. Initial stress condition (pre-adjusted) for the Limestone material.

Using equations (8) through (13), the model stresses in both the diorite and the limestone materials are adjusted to match the measured stresses. The adjusted stress state is shown in Figures 5 & 6.

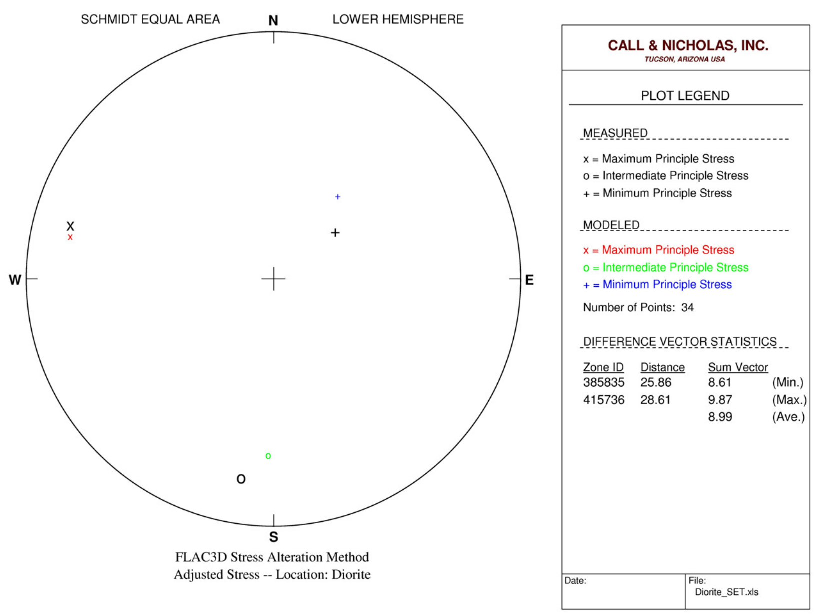

Figure 5. Diorite adjusted stresses.

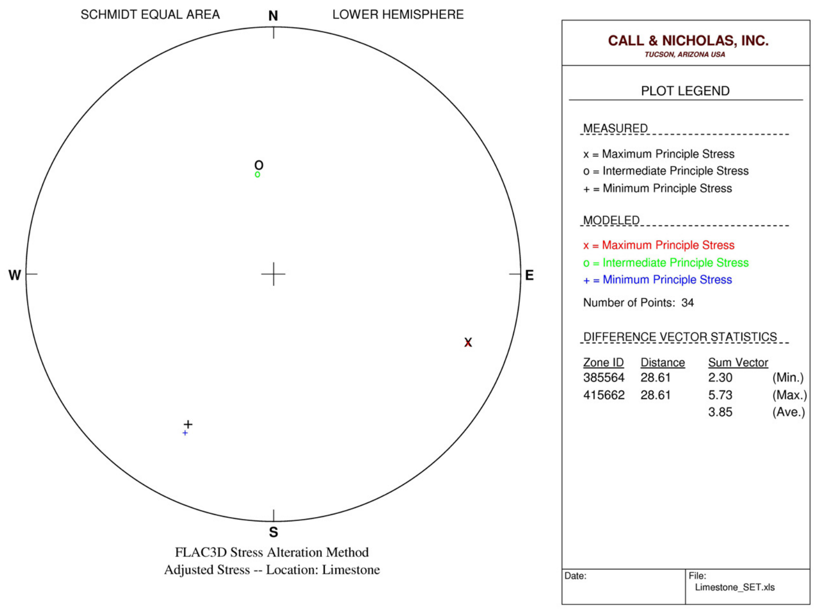

Figure 6. Limestone adjusted stresses.

Finally, the model is cycled to the final equilibrium condition and the fully equilibrated stresses are shown in Figures 7 & 8. Some deviation from the set stresses are expected as the model adjusts, however in addition to the Schmidt plot now showing a good correlation to the measured orientations, the SDV calculation shows that there has been an improvement in the total error, indicating that the stress magnitudes are in agreement as well.

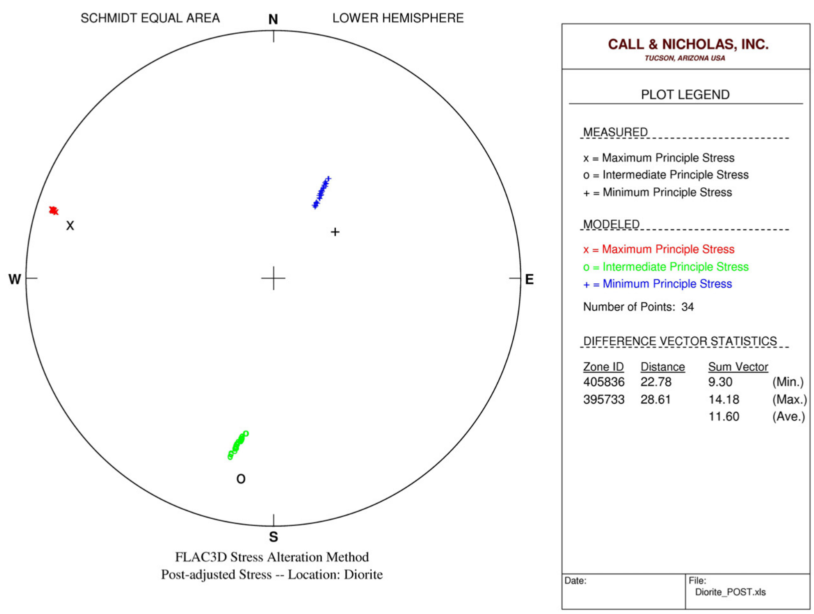

Figure 7. Final stress condition (post-adjusted) for the Diorite material.

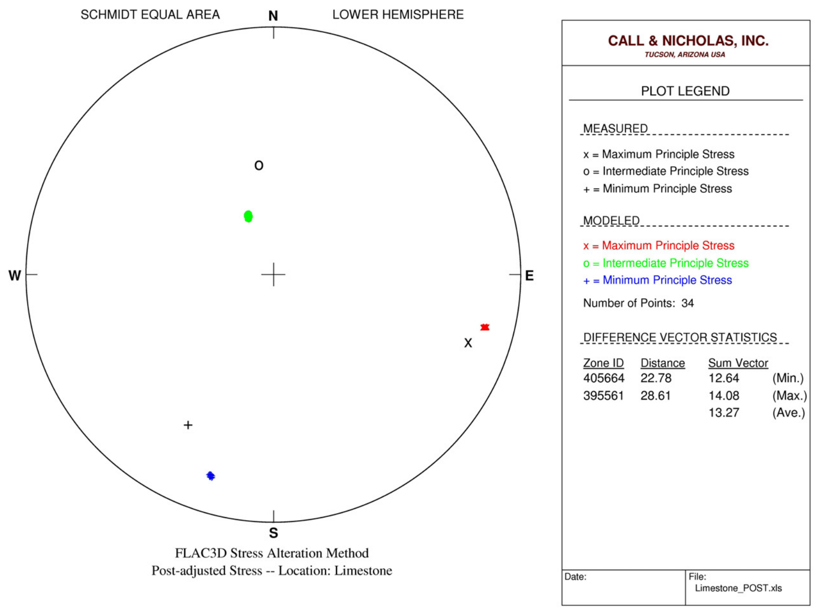

Figure 8. Final stress condition (post-adjusted) for the Limestone material.

10 CONCLUSIONS

Initialization of a specific non-uniform stress state in a *FLAC3D* grid is a valid method of increasing the performance and capability of a numerical model simulation. When a specific non-uniform stress state is fit to field-verified stress measurement data, the reliability of the numerical model is greatly enhanced.

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