

An Analysis to Compare Factor of Safety Values Between the Limit Equilibrium Method and Shear Strength Reduction Technique

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ABSTRACT: Computer modeling for large-scale rock slopes has become essential to assess factor of safety (FOS) values to predict slope instability and estimate potential failure. The limit equilibrium method (LEM) provides FOS values according to force and moment equilibrium; the shear strength reduction (SSR) technique calculates FOS using stress- and deformation-based analyses. Currently, both methods are used by geotechnical engineers to analyze large-scale rock slopes for mining and civil engineering projects. Both methods can be used simultaneously to check critical slip surface locations and FOS values.

This study was performed to understand similarities and differences in FOS calculations between the two computer modeling programs for high rock slopes greater than 600 meters. The FOS values obtained indicate a SSR generates results roughly 0.1 lower than the traditional LEM when dealing with homogeneous rock slope geometry. However, when additional geology is included within the rock slope, resulting FOS values between the two methods may be higher or lower depending on the modeling program limits. Several sensitivities were run using different pore pressure conditions, *in situ* stress ratios, and geologic conditions. The results generated from the sensitivities and the conclusions are presented in this study.

1. INTRODUCTION

In the mining and civil engineering industries, slope stability analyses have become essential to ensure site safety, maximize ore removal, and limit interruptions to production. Slope geometry, material properties, and *insitu* stress conditions need to be investigated in order to understand the potential for rock slides, wedge failures, and other instabilities. Numerical programs are available to assist geotechnical engineers in several tasks such as estimating the factor of safety (FOS) for the stability of the road cut or mine slopes, as well as predicting displacements and stresses.

Care should be taken however as these programs will all compute different results and usually produce varying

individual slip surfaces. The factor of safety values obtained from these programs must be carefully analyzed with the realization that the limit equilibrium method (LEM) and the shear strength reduction technique (SSR) may produce FOS values different from one another. As noted, Stead et al (2006) [1] recommends combining the use of both the LEM and numerical modeling techniques to ensure maximum certainty and utilize the advantages to both methods.

Previous studies have presented comparisons of FOS values for small-scale soil or rock slopes, notable examples are found in Hammah et. al. (2005), Cala & Flisiak (2001) and Cala & Flisiak (2004) [2,3,4]. These papers show that the factor of safety values computed for both LEM and SSR are reasonable and generally agree. Though these findings have been very useful for smaller

scale civil engineering work, more research is needed on the applicability of both methods for large scale rock slopes with heights over 600 meters, typical for rock slopes in mining operations.

This paper will investigate the FOS values obtained through the use of LEM and SSR techniques for large scale rock slopes. Both homogeneous and heterogeneous materials are considered as well as the influence of insitu horizontal stress and water. This study shows how the FOS values compare when using a stress based numerical model (SSR) technique to a moment/force based method (LEM) technique.

2. METHODOLOGY

Various types of scenarios were constructed using two modeling programs containing the LEM or SSR method. Three models were built for this study. The first was a simple one material, 10 meter high soil slope. This was constructed to verify the results of previous studies. The second model was a single material, 915 meter high rock slope. The last model was a multiple material 915 meter high rock slope having at least three materials and at most five materials. For each model the horizontal to vertical stress ratio (k_0) varied from 0.33 to 1.0 and contained both dry and wet scenarios. An overall slope angle of 45 degrees (1:1 ratio) was used for all models. The friction angle and cohesion pairs which are derived for each LEM scenario were initially estimated using the rock engineering charts shown in Hoek & Bray (1981) [5]. Wet condition number four was utilized to determine the phreatic surface for wet conditions.

Minor adjustment of the cohesive strength was performed until the LEM produced FOS values of exactly 1.2, 1.1, and 1.0 for each scenario. These resulting strength values (friction angle and cohesion) were then input into separate numerical models for calculation of the SSR method FOS value.

This investigation computed LEM FOS values using Geostudios Slope/W [6] and SSR technique FOS values using Itasca's Fast Lagrangian Analysis of Continua (FLAC) [7]. A Mohr-Coloumb perfectly plastic yield criteria was assumed.

2.1 Limit Equilibrium Method (LEM)

The Limit Equilibrium Method (LEM) is based on static force and moment equilibrium. The method has been in widespread use since the early 20th century and may be considered the oldest method used for stability calculations in geological engineering.

During this investigation the Spencer Method [8], a method of slices that satisfies both force and moment equilibrium, will be used.

The factor of safety is calculated as a ratio of resisting forces to driving forces and expressed in the following format:

$$FOS = \frac{\text{Resisting Forces}}{\text{Driving Forces}} \quad (1)$$

$$FOS = \frac{cA + W \cos \psi \tan \phi}{W \sin \phi} \quad (2)$$

Where; c is cohesion, ψ is the angle of the slope, ϕ is the friction angle of the rock material and W is the weight of the rock mass in question. When the cohesion of the material is not present in equation (2) and the slope is dry, the rock block will slide when the dip angle of the sliding surface equals the friction angle of the rock [9]. Stability is independent of any deformation or the size of the sliding block, and the driving forces equal the resisting forces indicating a FOS of 1.0. A FOS of 1.0 is assumed to be the "threshold" where the rock slope goes from stable to unstable. When the FOS is less than 1.0, the driving forces overcome the resisting forces causing slope movement.

2.2 Shear Strength Reduction Technique (SSR)

The limit equilibrium method presents a conventional procedure to obtain the factor of safety for the critical slip surface in the rock slope; however, FLAC utilizes an alternative method to calculate the factor of safety. The traditional definition of the factor of safety for slopes stability analysis is to calculate the factor of safety with respect to the soil/rock shear strength [10]. The factor of safety of a slope can be computed with a finite element or finite difference code by reducing the estimated rock shear strength in stages until the slope fails. The resulting factor of safety is the ratio of the estimated shear strength to the reduced shear strength at failure. This method is called the shear strength reduction technique and is described by Dawson et al (1999) [11]. The equations used for the shear strength reduction (SSR) are the following:

$$c_{\text{trial}} = (1/F) \quad (3)$$

$$\phi_{\text{trial}} = \arctan\{(1/F) \tan \phi\} \quad (4)$$

where; c is cohesion, ϕ is the friction angle, and F is the factor of safety. During the reduction process, the trial parameters will keep changing until the factor of safety satisfies equations 3 and 4. The FLAC program may err when the FOS falls well below 1.0, failing to calculate a FOS. When this occurred during a simulation the FOS was reported to be less than 0.95, indicating failure.

3 HOMOGENEOUS SOIL SLOPE

A model of a 10 meter high slope consisting of a single material with a unit weight of 19.6 kN/m³ was constructed. Figure 1 shows the Slope/W model that was created. A typical slip surface obtained for this model is illustrated for reference.

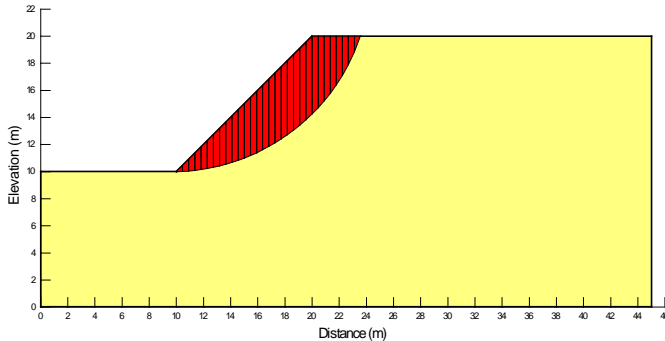


Figure 1: 10 meter high soil slope, FOS = 1.0

3.1 Material Properties

Table 1 reports the material properties derived for the 10 meter homogeneous soil slope model to obtain FOS values of 1.2, 1.1, and 1.0 for the LEM. These material properties were then input into a FLAC model which exactly matched the Slope/W model.

Table 1: Derived Material Strengths for 10 Meter Slope

FOS	Dry Condition		1/2 Saturated	
	Phi (deg)	Coh (kPa)	Phi (deg)	Coh (kPa)
1.3	21	19.6	26	24.2
1.2	19	18.6	24	21.0
1.1	17	17.5	22	19.4
1.0	15	16.4	20	17.7

3.2 Factor of Safety Results

Table 2 compares the factor of safety values returned for the 10 meter homogeneous soil slope.

Table 2: FOS Comparison for 10 Meter Slope
 $k_o = 1$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.29	1.30	1.26
1.2	1.20	1.19	1.20	1.19
1.1	1.10	1.09	1.10	1.09
1.0	1.00	1.00	1.00	1.00

$k_o = 0.33$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.29	1.30	1.27

1.2	1.20	1.19	1.20	1.19
1.1	1.10	1.09	1.10	1.09
1.0	1.00	1.00	1.00	0.99

4 HOMOGENEOUS ROCK SLOPE

A model of a 915 meter high slope consisting of a single material with a unit weight of 25.1 kN/m³ was constructed. Figure 2 shows the Slope/W model that was created. For illustration purposes, the slip surface obtained for a FOS value of 1.0 is shown.

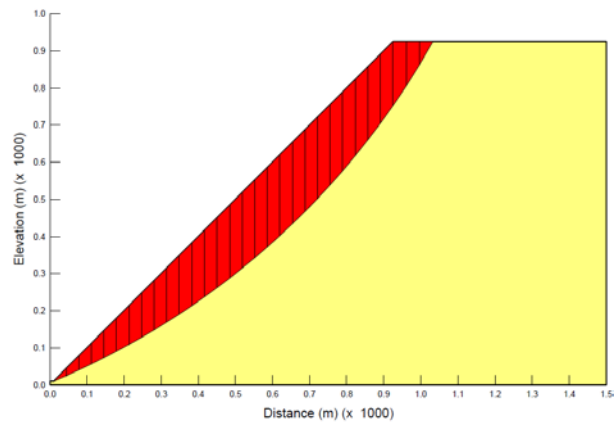


Figure 2: 915 meter high rock slope, FOS = 1.0

4.1 Material Properties

Table 3 reports the material properties derived for the 915 meter homogeneous rock slope model to obtain FOS values of 1.2, 1.1, and 1.0 for the LEM.

Table 3: Derived Material Strengths for 915 Meter High Slope

FOS	Dry Condition		1/2 Saturated	
	Phi (deg)	Coh (kPa)	Phi (deg)	Coh (kPa)
1.3	39	687	47	1177.9
1.2	37	632.0	43	1211.4
1.1	36	478.8	40	1137.2
1.0	35	344.7	36	1130.0

4.2 Factor of Safety Results

Table 4 compares the factor of safety values returned for the 915 meter homogeneous rock slope.

Table 4: FOS Comparison for 915 Meter Slope
 $k_o = 1$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.18	1.30	1.22
1.2	1.20	1.09	1.20	1.18

1.1	1.10	0.97	1.10	1.08
1.0	1.00	<0.95	1.00	0.97

$k_o = 0.33$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.22	1.30	1.22
1.2	1.20	1.12	1.20	1.10
1.1	1.10	1.01	1.10	1.03
1.0	1.00	<0.95	1.00	0.99

5 HETEROGENEOUS ROCK SLOPE

Real world modelling necessitates the use of multiple materials in numerical models. The heterogeneous rock slope example investigates the effect that multiple materials have on FOS estimations. All of the heterogeneous rock slope examples were built with a 915 meter high slope model.

5.1 Three Material Model

A three material model was constructed with three different rock layers with equal heights. Figure 4 shows the Slope/W model that was created. For illustration purposes, the slip surface obtained for a FOS value of 1.0 is shown.

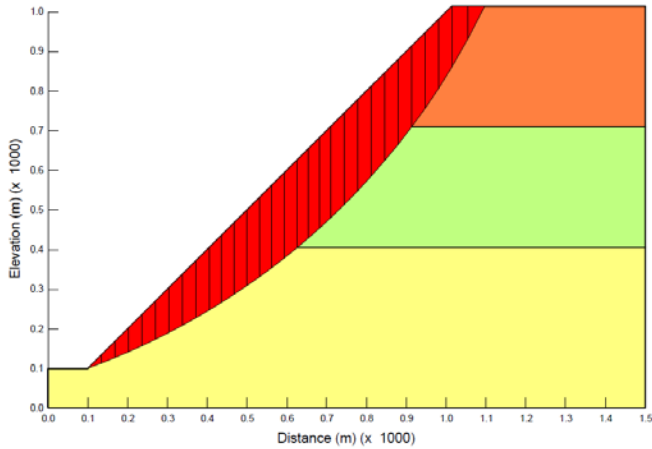


Figure 4: Three material model, FOS = 1.0

5.1.1 Material Properties

Table 7 reports the material properties derived for the 915 meter, three material type rock slope model, to obtain FOS values of 1.2, 1.1, and 1.0 for the LEM. Layer numbers start from the bottom.

Table 7: Derived Material Strengths for Three Material Model

Dry Condition	Factor of Safety			
	1.0	1.1	1.2	1.3
Material Properties				
Layer 1 Phi (deg)	34	37	40	43
Layer 1 C (kPa)	359.1	368.7	368.7	373.5
Layer 2 Phi (deg)	35	38	41	44
Layer 2 C (kPa)	383	392.6	392.6	383.0
Layer 3 Phi (deg)	36	39	42	45
Layer 3 C (kPa)	397.4	407	407	387.8

1/2 Saturated	Factor of Safety			
	1.0	1.1	1.2	1.3
Material Properties				
Layer 1 Phi (deg)	45	49	52	55
Layer 1 C (kPa)	574.6	560.2	574.6	790
Layer 2 Phi (deg)	46	50	53	57
Layer 2 C (kPa)	588.9	584.1	598.5	804.4
Layer 3 Phi (deg)	47	51	54	58
Layer 3 C (kPa)	598.5	588.9	608.1	837.9

5.1.2 Factor of Safety Results

Table 8 compares the factor of safety values returned for the three material rock slope model.

Table 8: FOS Comparison for Three Material Model

$k_o = 1$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.12	1.30	1.27
1.2	1.20	1.06	1.20	1.01
1.1	1.10	0.96	1.10	1.00
1.0	1.00	<0.95	1.00	<0.95

$k_o = 0.33$

FOS	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.17	1.30	1.26
1.2	1.20	1.08	1.20	1.03
1.1	1.10	0.97	1.10	0.99
1.0	1.00	<0.95	1.00	<0.95

5.2 Five Material Model

A five material model was constructed with five different rock layers with equal heights. Figure 6 shows the Slope/W model that was created. For illustration purposes, the slip surface obtained for a FOS value of 1.0 is shown.

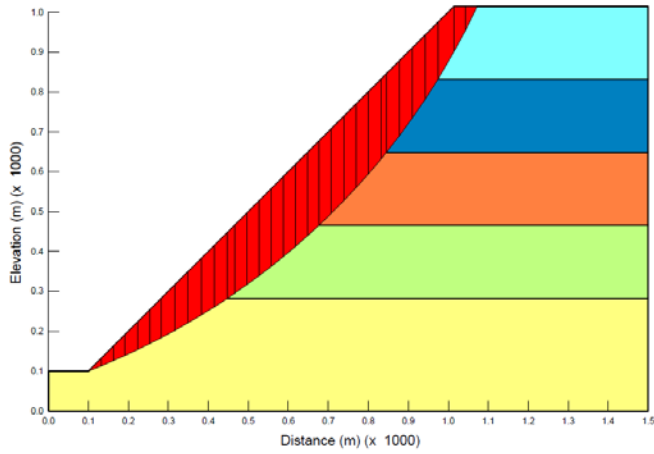


Figure 6: Five rock material model, FOS = 1.0

5.2.1 Material Properties

Table 11 reports the material properties derived for the 915 meter, four material type rock slope model, to obtain FOS values of 1.2, 1.1, and 1.0 for the LEM. Layer numbers start from the bottom.

Table 11: Derived Material Strengths for Five Material Model

Dry Condition	Factor of Safety			
Material Properties	1.0	1.1	1.2	1.3
Layer 1 Phi (deg)	34	37	40	43
Layer 1 C (kPa)	320.8	335.2	335.2	320.8
Layer 2 Phi (deg)	35	38	41	44
Layer 2 C (kPa)	325.6	344.7	339.9	330.4
Layer 3 Phi (deg)	36	39	42	45
Layer 3 C (kPa)	335.2	354.3	349.5	335.2
Layer 4 Phi (deg)	37	40	43	46
Layer 4 C (kPa)	339.9	363.9	359.1	344.7
Layer 5 Phi (deg)	39	41	44	47
Layer 5 C (kPa)	359.1	368.7	368.7	349.5

1/2 Saturated	Factor of Safety			
Material Properties	1.0	1.1	1.2	1.3
Layer 1 Phi (deg)	45	49	52	55
Layer 1 C (kPa)	541	526.7	541	780.4
Layer 2 Phi (deg)	46	50	53	56
Layer 2 C (kPa)	550.6	536.3	550.6	794.8
Layer 3 Phi (deg)	47	51	54	57
Layer 3 C (kPa)	569.8	545.8	560.2	799.6
Layer 4 Phi (deg)	48	52	55	58
Layer 4 C (kPa)	574.6	555.4	569.8	818.8
Layer 5 Phi (deg)	49	53	56	59
Layer 5 C (kPa)	584.1	560.2	579.3	828.3

5.2.2 Factor of Safety Results

Table 12 compares the factor of safety values returned for the five material rock slope model.

Table 12: FOS Comparison for Five Material Model

FOS	$k_o = 1$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.12	1.30	1.21
1.2	1.20	1.03	1.20	1.01
1.1	1.10	0.97	1.10	0.98
1.0	1.00	<0.95	1.00	<0.95

FOS	$k_o = 0.33$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.16	1.30	1.24
1.2	1.20	1.08	1.20	0.98
1.1	1.10	0.97	1.10	<0.95
1.0	1.00	<0.95	1.00	<0.95

6 DISCUSSION

Part of the aim of this study was to verify the results of previous studies which showed that for small scale slopes, the LEM and SSR technique computes FOS values which are reasonable and in agreement.

Table 13: FOS Comparison for 10 Meter Slope

FOS	$k_o = 1$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.29	1.30	1.26
1.2	1.20	1.19	1.20	1.19
1.1	1.10	1.09	1.10	1.09
1.0	1.00	1.00	1.00	1.00

FOS	$k_o = 0.33$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.29	1.30	1.27
1.2	1.20	1.19	1.20	1.19
1.1	1.10	1.09	1.10	1.09
1.0	1.00	1.00	1.00	0.99

The results of the 10 meter slope simulations show that for varying values of pore pressure and horizontal stress, the FOS value returned using the both the LEM and the SSR technique agree.

For slope heights that are typical in the mining industry, factor of safety values returned for the LEM and SSR technique diverge. When a 915 meter high rock slope was simulated, the SSR technique produced lower values than the LEM.

Table 14: FOS Comparison for 915 Meter Slope

FOS	$k_0 = 1$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.18	1.30	1.22
1.2	1.20	1.09	1.20	1.18
1.1	1.10	0.97	1.10	1.08
1.0	1.00	<0.95	1.00	0.97

FOS	$k_0 = 0.33$			
	Dry		1/2 Saturated	
	LEM	SSR	LEM	SSR
1.3	1.30	1.22	1.30	1.22
1.2	1.20	1.12	1.20	1.10
1.1	1.10	1.01	1.10	1.03
1.0	1.00	<0.95	1.00	0.99

The SSR method always produced lower values. In some cases, the SSR technique returns FOS values that are close to the LEM, and other cases return FOS values which are as much as 0.1 lower.

When heterogeneity is applied to the slope materials, the SSR technique returns more divergent results. As more materials are added to the model, the FOS decreases. The results also diverge as the model assumptions change, either with the horizontal stress, or with the addition of pore pressure.

These differences in FOS can be attributed directly to the calculation methods. The FOS in LEM is simply the ratio of the resisting forces to driving forces. For the SSR technique, the FOS is the ratio of the strength at failure versus the initial estimated strength. At low magnitude normal stresses, as in the case of the 10 meter slope model, the two methods produce similar results. As the slope heights increase, normal stresses increase in the FLAC model and the results diverge.

This can be graphically exhibited in a normal-shear stress plot. The normal and shear stress obtained directly from a particular FLAC model zone is plotted versus the calculated normal and shear stress of a matching location at the base of a slice in the LEM model.

Figures 7 and 8 present an example of each case, where the slope height is 10 meters and where the slope height is 915 meters.

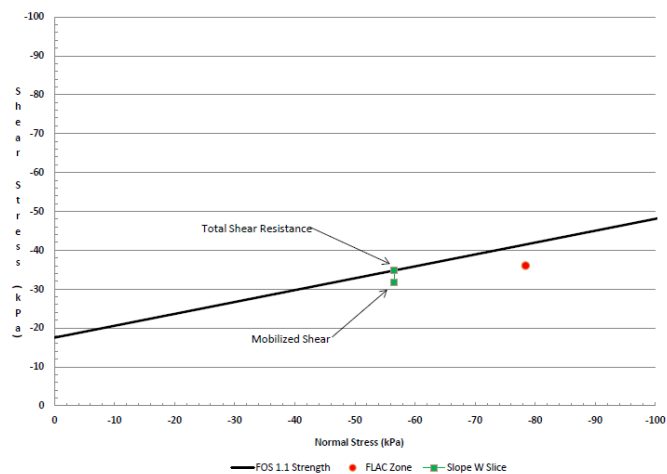


Figure 7: Stress conditions for 10 m homogeneous soil slope, FOS = 1.1

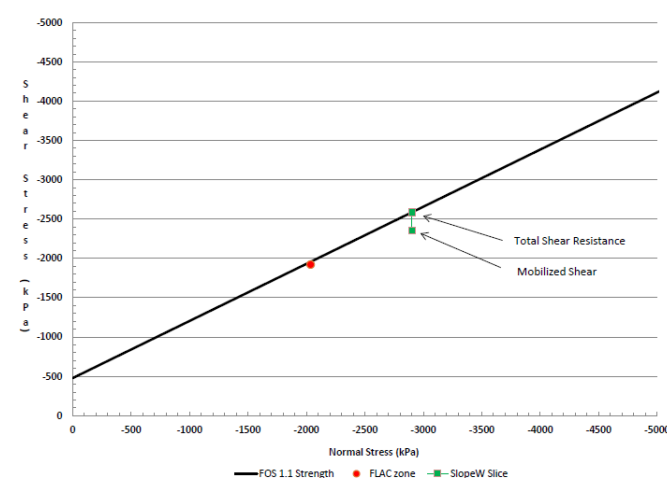


Figure 8: Stress conditions for 915 m homogeneous rock slope, FOS = 1.1

7 CONCLUSIONS

The results obtained from this study have shown that for small scale slopes, the LEM and SSR technique computes FOS values which are in agreement. For high slopes, the SSR method will always produce a lower FOS value than the LEM and will generally produce a FOS value that is 0.1 lower than for LEM.

In summary, caution is advised when running one of the two methods for estimating the factor of safety. Both LEM and SSR should be considered when analyzing large-scale rock slopes for mining and civil engineering projects.

8 REFERENCES

1. Stead, D., E. Eberhardt, J. Coggan. 2006. Developments in the characterization of complex rock slope deformation and failure using numerical

- modeling techniques. *Engineering Geology* pp 217-235, Vol 83.
2. Hammah, R.E., T. Yacoub, B. Corkum, J. Curran. 2005. A comparison of finite element slope stability analysis with conventional limit-equilibrium investigation. *Proceedings of the 58th Canadian Geotechnical and 6th Joint IAHC- CNC and CGS Groundwater Specialty Conferences Saskatoon, Saskatchewan, Canada, September 2005*
 3. Cala M. & J. Flisiak. 2001. Slope stability analysis with FLAC and limit equilibrium methods. In Bilaux, Rachez, Detournay & Hart (eds.) *FLAC and Numerical Modelling in Geomechanics*: 111-114. A.A. Balkema Publishers.
 4. Cala, M. & J. Flisiak. 2003. Complex geology slope stability analysis by shear strength reduction. In Brummer, An-drieux, Detournay & Hart (eds.) *FLAC and Numerical Modeling in Geomechanics – 2003, Proceedings of the 3rd International Symposium, Sudbury, Ontario*.
 5. Hoek, E., J. Bray. 1981. *Rock Slope Engineering, Revised 3rd Edition*. Taylor & Francis, New York,
 6. Geo-slope International, 2007. Geostudio Software: Slope/W. Calgary, Alberta, Canada.
 7. Itasca, 2006. Fast Lagrangian Analysis of Continua (FLAC). Version 5.0. Minneapolis, MN.
 8. Spencer, E. 1967. A Method of Analysis of the Stability of Embankments Assuming Parallel Inter-Slice Forces. *Geotechnique* vol 17 pp 11-26.
 9. Wyllie, D., C. Mah. 2004. *Rock Slope Engineering: Civil and Mining, 4th Edition*. Spon Press, Taylor & Francis Group, London and New York.
 10. Sainsbury, D., M. E. Pierce and L. J. Lorig. 2003. Two- and Three-Dimensional Numerical Analysis of the Interaction Between Open-Pit Slope Stability and Remnant Underground Voids. *Harnessing Technology by Managing Data and Information (5th Large Open Pit Conference, Kalgoorlie, Western Australia, November 2003)*, pp. 251- 257, C. Workman-Davies and E. Chanda, Eds. Carlton: AUSIMM.
 11. Dawson, E., W. Roth, A. Drescher. 1999. Slope stability analysis by strength reduction. *Geotechnique* Vol 49, No. 6 pp 835-840. NY reprint 1999. The Institution of Mining & Metallurgy, 1974.